Math 162: Calculus IIA

Second Midterm Exam

November 11, 2021

NAME (please print legibly): ____________________________

Your University ID Number: ____________________________

Your University email ____________________________

Indicate your instructor with a check in the box:

Bogdan Krstic	MW 9:00 - 10:15 AM	
Doug Ravenel	MWF 10:25 - 11:40 AM	
Charles Wolf	MW 12:30 - 1:45 PM	

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam and that all work will be my own.

Signature: ____________________________

- **The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden. IF YOU HAVE YOUR PHONE WITH YOU, YOU MUST KEEP IT OUT OF REACH OR TURN IT IN TO A PROCTOR BEFORE STARTING THE EXAM. FAILURE TO DO SO WILL BE TREATED AS AN ACADEMIC HONESTY VIOLATION.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given. If some of your work is not on the page where the problem appears, indicate where it is.**
- **Put your answers in the space provided at the bottom of each page.**
- **You are responsible for checking that this exam has all 12 pages.**

Handy dandy formulas

Integration by parts formula:

$$\int u\,dv = uv - \int v\,du$$

Trigonometric identities:

$$\cos^2\theta + \sin^2\theta = 1 \qquad \sec^2\theta - \tan^2\theta = 1$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2} \qquad \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

Derivatives of trig functions.

$$\frac{d\sin x}{dx} = \cos x \qquad \frac{d\tan x}{dx} = \sec^2 x \qquad \frac{d\sec x}{dx} = \sec x\tan x$$

$$\frac{d\cos x}{dx} = -\sin x \qquad \frac{d\cot x}{dx} = -\csc^2 x \qquad \frac{d\csc x}{dx} = -\csc x\cot x$$

Trigonometric substitution for integrals of the form

$$\int \tan^m x\sec^n x\,dx \qquad \text{with } n > 0,$$

known in Doug's section as *the rabbit trick.*

$$u = \sec x + \tan x \qquad \sec x\,dx = \frac{du}{u}$$

$$\sec x = \frac{u^2+1}{2u} \qquad \tan x = \frac{u^2-1}{2u}$$

Area of surface of revolution in rectangular coordinates, $y = f(x)$ with $a \le x \le b$

- about the x-axis: $$S = 2\pi\int_a^b f(x)\sqrt{1+f'(x)^2}\,dx$$
- about the y-axis: $$S = 2\pi\int_a^b x\sqrt{1+f'(x)^2}\,dx$$

More formulas for your enjoyment

Polar coordinates

$$r = \sqrt{x^2 + y^2} \qquad \theta = \begin{cases} \arctan(y/x) & \text{for } x > 0 \\ \pi + \arctan(y/x) & \text{for } x < 0 \\ \pi/2 & \text{for } x = 0 \text{ and } y > 0 \\ 3\pi/2 & \text{for } x = 0 \text{ and } y < 0 \\ \text{undefined} & \text{for } (x, y) = (0, 0) \end{cases}$$

$$x = r\cos\theta \qquad y = r\sin\theta$$

Changing θ by any multiple of 2π does not change the location of the point. Changing the sign of r is equivalent to adding π to θ, which is the same as moving the point to one in the opposite direction and the same distance from the origin.

Area in polar coordinates for $r = f(\theta)$ with $\alpha \leq \theta \leq \beta$:

$$A = \int_\alpha^\beta \frac{r^2}{2}\, d\theta$$

Arc length formulas

- Rectangular coordinates, $y = f(x)$ with $a \leq x \leq b$:

$$S = \int_a^b \sqrt{1 + f'(x)^2}\, dx$$

- Polar coordinates, $r = f(\theta)$ with $\alpha \leq \theta \leq \beta$:

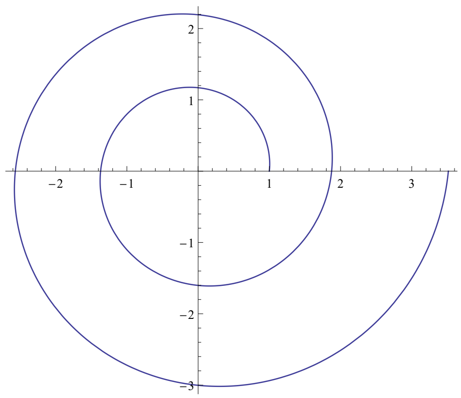
$$S = \int_\alpha^\beta \sqrt{r^2 + f'(\theta)^2}\, d\theta$$

- Parametric equations, $x = x(t)$ and $y = y(t)$ with $a \leq t \leq b$:

$$S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

1. (20 points)

(a) (10 POINTS) Find the arc length of the polar curve $r = e^{k\theta}$ for $0 \leq \theta \leq 4\pi$ where $k = 1/10$, as shown below. You may express your answer in terms of e, π and $\sqrt{1.01}$.

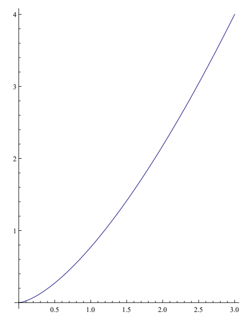


ANSWER:

(b) (10 POINTS) Find the arc length of the parametric curve

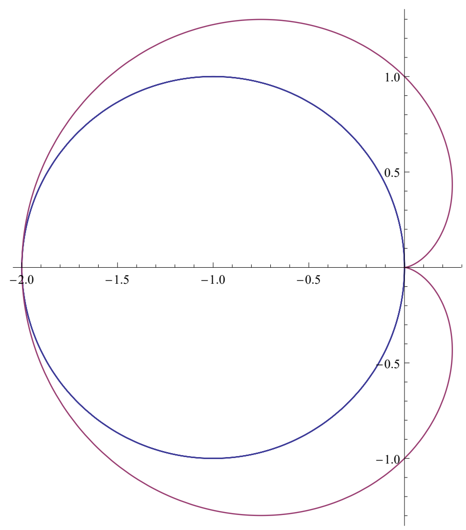
$$x = 3t^2 \qquad\qquad y = 4t^3$$

for $0 \leq t \leq 1$, as shown below. HINT: Your answer must exceed the length of the straight line joining the two endpoints, which is 5.



ANSWER:

2. (20 points) Find the area of the cardioid defined in polar coordinates by $r = 1 - \cos\theta$ for $0 \leq \theta \leq 2\pi$, shown below. Write your answer in terms of π. HINT: The cardioid contains a circle of radius 1, also shown below, so its area must exceed π.



ANSWER:

3. (20 points) Determine if each of the following sequences is convergent or divergent, and find the limit if the sequence is convergent. For full credit, your work must support your answers.

(a) (10 POINTS)

$$\{a_n\}_{n=1}^{\infty} = \left\{ \frac{n + \cos^2\left(\frac{1}{n}\right)}{n+1} \right\}_{n=1}^{\infty}$$

ANSWER:

(b) (10 POINTS)

$$\{b_n\}_{n=1}^{\infty} = \left\{n^{-1}e^n\right\}_{n=1}^{\infty}$$

ANSWER:

4. (20 points)

Compute the surface area of the solid formed by revolving $y = x^3$ for $0 \leq x < 1$ about the x-axis.

ANSWER:

5. (20 points) Compute the following indefinite integral:

$$\int \frac{dx}{(x-2)^2(x+1)}$$

ANSWER:

Scratch paper

More scratch paper