Math 162: Calculus IIA

Second Midterm Exam, Morning Edition

November 6, 2020

NAME (please print legibly): _____

Your University ID Number: _____

Your University email _____

Write the name of your proctor here.

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam and that all work will be my own.

Signature: _____

Instructions

- **You may not consult the textbook, your notes, the internet, your classmates, friends or any other external source of information. YOUR WEBCAM MUST BE ON AT ALL TIMES.**
- **If you have access to a printer, you may print this exam and write your answers in the spaces provided. Otherwise, write the answers to each problem on a separate sheet of paper. YOU MUST ALSO WRITE AND SIGN THE PLEDGE OF HONESTY AND GIVE ALL OF THE INFORMATION REQUESTED ABOVE.**
- **Show your work and justify your answers. You may use the formulas on the next page. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **You must finish work on this exam by 9:15, and then scan and upload it to Gradescope as previously instructed by 9:30. Exams received after that time will be subject to a penalty.**

Trig formulas:

- $\cos^2(x) + \sin^2(x) = 1$
- $\sec^2(x) - \tan^2(x) = 1$
- $\sin(2x) = 2\sin(x)\cos(x)$
- $\cos^2(x) = \dfrac{1 + \cos(2x)}{2}$
- $\sin^2(x) = \dfrac{1 - \cos(2x)}{2}$

Polar coordinate formulas:

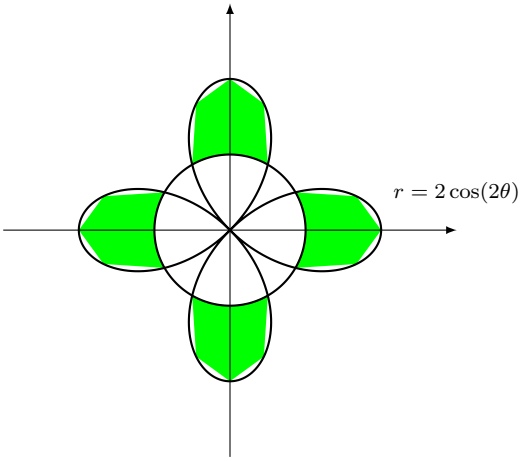
- Area:
$$\frac{1}{2}\int r^2 d\theta$$
- Arc length:
$$\int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

Parametric equation formulas:

- Newton's notation: $\dot{x} = dx/dt \qquad \dot{y} = dy/dt$
- Slope of tangent line: $dy/dx = \dot{y}/\dot{x}$.
- Second derivative
$$\frac{d^2y}{dx^2} = \frac{d(\dot{y}/\dot{x})/dt}{\dot{x}}.$$
Curve is concave up/down when this is positive/negative.
- Arc length:
$$\int \sqrt{\dot{x}^2 + \dot{y}^2}\, dt$$

1. (25 points)

Compute the area inside the polar curve $r = 2\cos(2\theta)$, a four leafed rose, and outside $r = 1$, a circle.



ANSWER:

2. (25 points)

Let C be the upper half of the circle centered at the origin with radius 3. The arc on C between $(0, 3)$ and $(\sqrt{5}, 2)$ is rotated about the x-axis to produce a surface S.

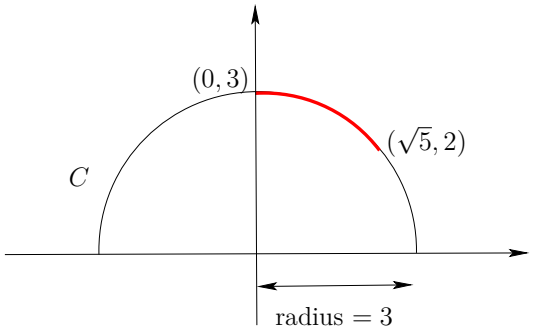


Figure 1:

(a) (15 points) Use

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

to find the surface area.

ANSWER:

(b) (10 points) Consider the same surface as in part (a). This time, use

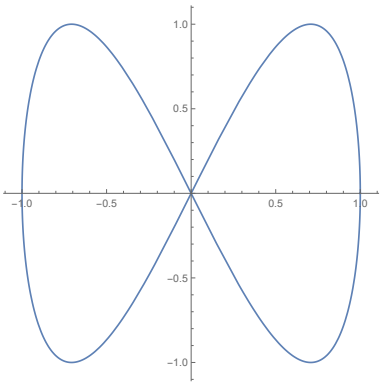
$$ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

to find the surface area.

ANSWER:

3. (25 points) Consider the parametric curve

$$x = \cos(t), \qquad y = \sin(2t), \qquad 0 \leq t \leq 2\pi$$



(a) (9 points) At what points is the tangent horizontal or vertical?

ANSWER:

(b) (8 points) The curve passes through the origin twice. What are the slopes of the two tangent lines to the curve at the origin?

ANSWER:

(c) (8 points) Find the equation of the form $y = mx + b$ for the tangent at $t = \frac{\pi}{6}$.

ANSWER:

4. (25 points)

Consider the logarithmic spiral $r = e^\theta$, $\theta \in [0, \infty)$, which can be defined parametrically by $x = e^t \cos t$ and $y = e^t \sin t$ with $t = \theta$.

(a) (13 points) Calculate the arc-length of the logarithmic spiral for $0 \leq \theta \leq b$.

ANSWER:

(b) Calculate the area between the curve and the x-axis for $\theta \in [0, \pi]$.

ANSWER: