Math 162: Calculus IIA

Second Midterm Exam

November 17, 2016

NAME (please print legibly): ______________________________

Your University ID Number: ______________________________

Your University email ______________________________

Indicate your instructor with a check in the box:

Jie Zhong	MWF 9:00 - 9:50 AM	
Doug Ravenel	MWF 10:25 - 11:15 AM	
Doug Haessig	MW 12:30 - 1:45 PM	
Carl McTague	MW 4:50-6:05 PM	

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam and that all work will be my own.

Signature: ______________________________

- **The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden. IF YOU HAVE YOUR PHONE WITH YOU, YOU MUST TURN IT IN TO A PROCTOR BEFORE STARTING THE EXAM. FAILURE TO DO SO WILL BE TREATED AS AN ACADEMIC HONESTY VIOLATION.**

- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**

- **Put your answers in the space provided at the bottom of each page or half page.**

- **You are responsible for checking that this exam has all 13 pages.**

QUESTION	VALUE	SCORE
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL	100	

1. (20 points)

The following are convergent series. Find the sum of each of them. **You don't need to justify the convergence of the series, but you should clearly show how you got the answer.**

(a)

$$\sum_{n=1}^{\infty} \frac{3}{4n^2+4n-3} = \sum_{n=1}^{\infty} \frac{3}{(2n+3)(2n-1)}$$

ANSWER:

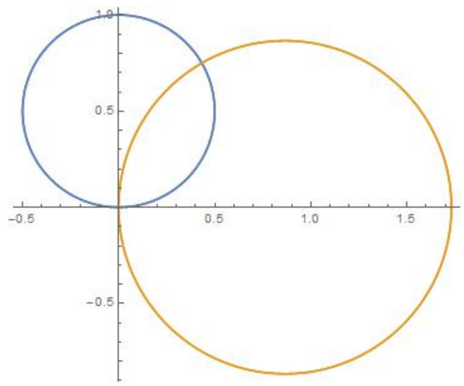
(b)

$$\sum_{n=2}^{\infty}(-2)^{n+1}3^{-n}$$

ANSWER:

2. (20 points)

(a) Find the area of the region both inside the circle $r = \sin\theta$ and outside the circle $r = \sqrt{3}\cos\theta$ (both equations are in polar coordinates). The two circles are shown below. THEY INTERSECT AT THE ORIGIN AND THE POLAR POINT $(\theta, r) = (\pi/3, \sqrt{3}/2)$.



ANSWER:

(b) Compute the equation (in Cartesian coordinates x, y) of the tangent line to the circle $r = \sin\theta$ at the points where it intersects the circle $r = \sqrt{3}\cos\theta$

ANSWER:

3. (20 points)

Determine whether the following series converge or diverge. Justify your answers, making sure to name the convergence test(s) that you are using.

(a)

$$\frac{1}{5}+\frac{1}{8}+\frac{1}{11}+\frac{1}{14}+\frac{1}{17}+\frac{1}{20}+\cdots$$

ANSWER:

(b)

$$\sum_{n=2}^{\infty} \frac{2}{n(\ln(n))^2 + 1}$$

.

ANSWER:

4. (20 points)

(a) Compute the area of surface of revolution obtained by rotating the curve $y = x^3$, for $0 \leq x \leq 1$, about the x-axis.

ANSWER:

(b) Do the same for the curve $y = |x|$, for $-1 \leq x \leq 1$.

ANSWER:

5. (20 points)

Find the arc-length of the parametric curve

$$x = \cos t - \cos 5t\,,\ y = \sin t - \sin 5t\,,\ 0 \leq t \leq 2\pi\,.$$

by doing it for $0 \leq t \leq \pi/2$ and multiplying your answer by 4.

YOU MAY WANT TO USE THE TRIG IDENTITIES $\cos(\alpha - \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ AND $\sin^2\theta = (1 - \cos 2\theta)/2$.

PROFESSORIAL SCREWUP NOTICE: The problem as stated above has a typo that makes it unreasonably difficult. In consideration for this error, anyone who atttempted it will be given full credit. The intended problem is stated below, and the picture matches it.

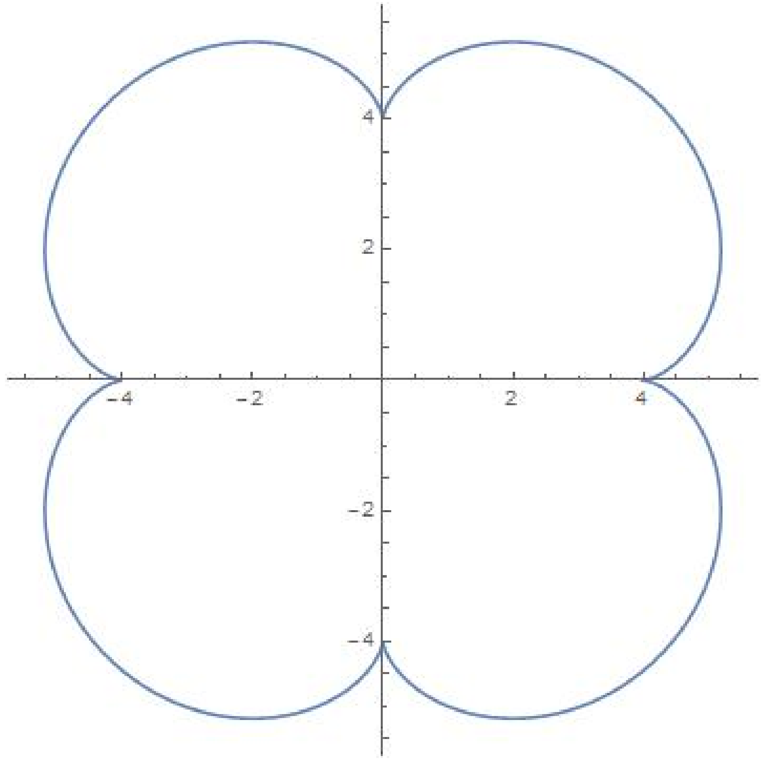
Find the arc-length of the parametric curve

$$x = 5\cos t - \cos 5t\,,\ y = 5\sin t - \sin 5t\,,\ 0 \leq t \leq 2\pi\,.$$

by doing it for $0 \leq t \leq \pi/2$ and multiplying your answer by 4.

YOU MAY WANT TO USE THE TRIG IDENTITIES $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ AND $\sin^2\theta = (1 - \cos 2\theta)/2$.

The curve for $0 \leq t \leq 2\pi$ is pictured below.


4
2
-4
-2
2
4
-2
-4

ANSWER: