Math 162: Calculus IIA

Second Midterm Exam

November 14, 2013

NAME (please print legibly): _____

Your University ID Number: _____

Indicate your instructor with a check in the box:

Yoonbok Lee	MWF 9:00 - 9:50 AM	Harkness 115	
Kalyani Madhu	TR 12:30-13:45 PM	Harkness 115	
Doug Ravenel	MWF 10:00 - 10:50 AM	Lattimore 201	
Geordie Richards	MWF 10:00 - 10:50 AM	Meliora 203	

- **The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden. IF YOU HAVE YOUR PHONE WITH YOU, YOU MUST TURN IT IN TO A PROCTOR BEFORE STARTING THE EXAM. FAILURE TO DO SO WILL BE TREATED AS AN ACADEMIC HONESTY VIOLATION.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **Put your answers in the space provided at the bottom of each page or half page.**
- **You are responsible for checking that this exam has all 11 pages.**

QUESTION	VALUE	SCORE
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL	100	

1. (20 points) Consider the curve

$$f(x) = \frac{x^4}{16} + \frac{1}{2x^2}.$$

(a) Calculate the arc length function $s(t)$ starting at $x = 1$, that computes the length of the curve from $(1, f(1))$ to $(t, f(t))$.

ANSWER:

(b) Calculate the arc length from $x = 2$ to $x = 4$.

ANSWER:

2. (20 points)

(a) Find the area of the surface of revolution obtained by rotating the curve $y = x^2$, for $0 \leq x \leq 2$, about the y-axis.

ANSWER:

(b) Find the area of the surface of revolution obtained by rotating the curve $x = 1 + |y|$, for $-1 \leq y \leq 1$, about the y-axis.

ANSWER:

3. (20 points) Determine whether the following series converge or diverge. Justify your answers, making sure to name the convergence test(s) that you are using.

(a)

$$\sum_{n=1}^{\infty} \frac{3^n + 1}{2^n - 1} = 4 + \frac{10}{3} + \frac{28}{7} + \frac{82}{15} + \cdots$$

ANSWER:

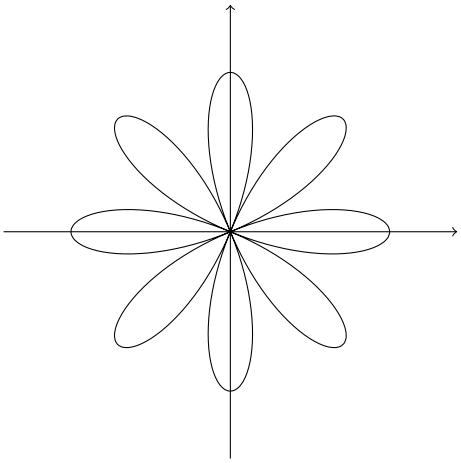
(b)

$$\sum_{n=2}^{\infty} \frac{1}{n \ln(n)^3} = \frac{1}{2 \ln(2)^3} + \frac{1}{3 \ln(3)^3} + \frac{1}{4 \ln(4)^3} + \cdots$$

ANSWER:

4. (20 points)

(a) Find the area of **one petal** of the polar rose $r = 2\cos(4\theta)$ pictured below.



ANSWER:

(b) The parametric curve given by $x = 4t^3 - 3t$, $y = t^2 + 1$ intersects the y-axis at 3 different values of t. What are the **equations of the tangent lines** to the curve at each of these points?

ANSWER:

5. (20 points) Find the sum of each of the following series.

(a)

$$\sum_{n=2}^{\infty} \frac{2}{n^2-1} = \frac{2}{3} + \frac{2}{8} + \frac{2}{15} + \frac{2}{24} + \frac{2}{35} + \cdots$$

HINT: USE PARTIAL FRACTIONS.

ANSWER:

(b)

$$\sum_{n=0}^{\infty} \left(\frac{1}{6 + (-1)^n} \right)^n = 1 + \frac{1}{5} + \frac{1}{7^2} + \frac{1}{5^3} + \frac{1}{7^4} + \cdots$$

Hint: Consider the evenly and oddly indexed terms separately.

ANSWER: