Math 162: Calculus IIA

Second Midterm Exam
November 10, 2011

NAME (please print legibly): _____

Your University ID Number: _____

Indicate your instructor with a check in the box:

Don Larson	MWF 9:00 - 9:50 AM	
Doug Ravenel	MWF 10:00 - 10:50 AM	
Yoonbok Lee	MWF 11:00 - 11:50 AM	

- The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.

- Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.

- Put your answers in the boxes provided at the bottom of each page or half page. *You will not get credit for answers written elsewhere.*

- You are responsible for checking that this exam has all 9 pages.

QUESTION	VALUE	SCORE
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL	100	

1. (20 points)

(a) Compute the area of surface of revolution obtained by rotating the curve $y = x^3$, for $0 \le x \le 1$, about the x-axis.
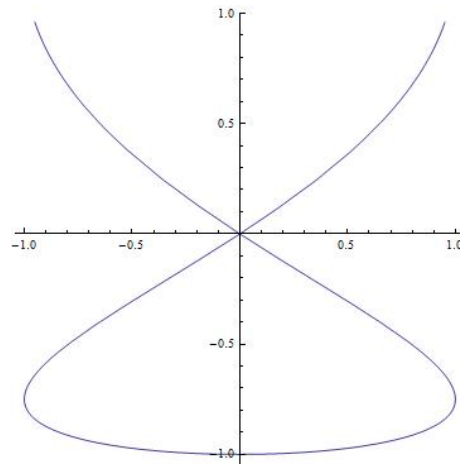
ANSWER:

(b) Do the same for the curve $y = |x|$, for $-1 \le x \le 1$.

ANSWER:

2. (20 points)

Consider the parametric curve $x = \sin(\pi t)$, $y = t^2 + 2t$, shown below for $-2.4 \le t \le .4$.

(a)   At what point(s) is the tangent line horizontal? *Do not use the picture to justify your answer!*

ANSWER:

(b)   The curve passes through the origin twice. What are the slopes of the two tangent lines to the curve at the origin?
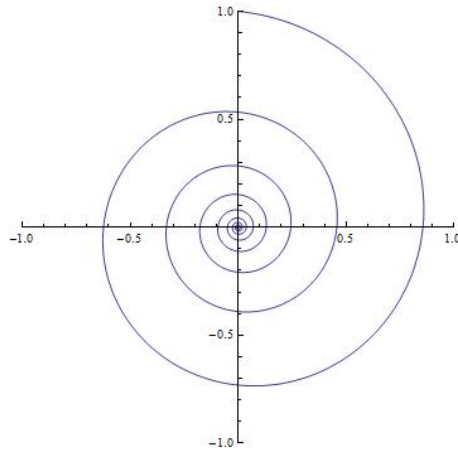
ANSWER:

(c)   Find the equation of the form $y = mx + b$ for the tangent at $t = 1$. (This point is not shown in the picture above.)

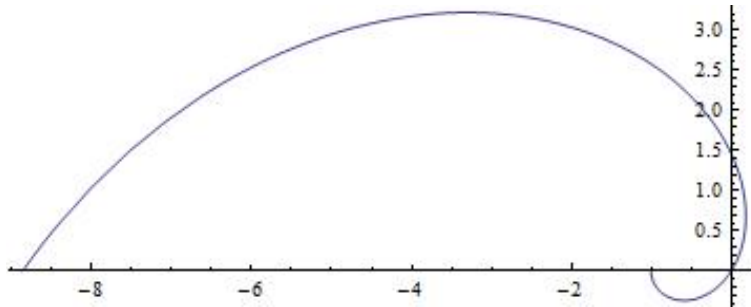| |
|---|
| ANSWER: |

3. (20 points)

Find the arc length of the parametric curve $x = e^{-t}\sin(10t)$, $y = e^{-t}\cos(10t)$, $0 \leq t < \infty$, which is the spiral shown below.
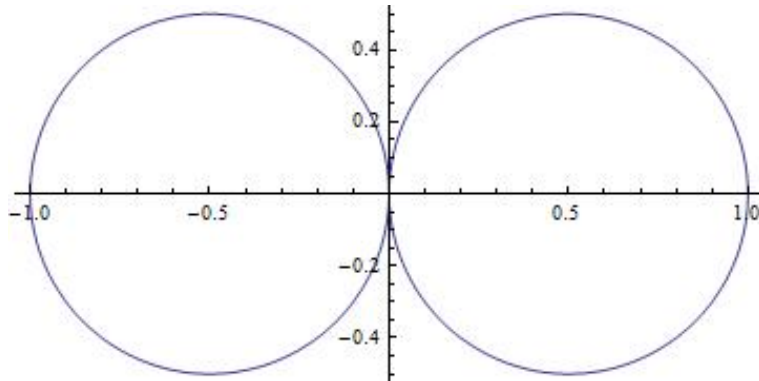

ANSWER:

4. (20 points)

(a)   Calculate the arc length of the curve $r = \theta^2 - 1$ for $0 \le \theta \le \pi$, which is shown below.

ANSWER:

(b)   *Use the polar coordinates area formula* to calculate the area enclosed by the curve $r = |\cos\theta|$ for $0 \leq \theta \leq 2\pi$, which is the "figure eight" shown below.

ANSWER:

5. (20 points)

(a) (5 points)   Does the sequence $\{a_n : n \geq 1\}$ with $a_n = 1/\sqrt[3]{n}$ converge? Why or why not?

ANSWER:

(b) (5 points)   Use L'Hôpital's Rule to show that for $k > 0$,

$$\lim_{x \to \infty} x^k \, e^{-x} = k \lim_{x \to \infty} x^{k-1} \, e^{-x} \, .$$

ANSWER:

c) (5 points)   Let $b_n = n^3 \, e^{-n}$. Show that the sequence $\{b_n : n \geq 1\}$ converges. What is the limit?

ANSWER:

(d) (5 points)   Does the sequence $\{c_n : n \geq 1\}$ with

$$c_n = \left(-\frac{1}{2}\right)^n \cos\left(\frac{n\pi}{3}\right)$$

converge? Why or why not?

ANSWER: