Math 162: Calculus IIA

Second Midterm Exam
November 11, 2010

NAME (please print legibly): _____

Your University ID Number: _____

Indicate your instructor with a check in the box:

Ang Wei	MWF 9:00 - 9:50 AM	
Doug Ravenel	MWF 10:00 - 10:50 AM	
Jon Carsteal	MW 2:00 - 3:15 PM	

- The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.

- Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.

- Put your answers in the boxes provided at the bottom of each page or half page. *You will not get credit for answers written elsewhere.*

- You are responsible for checking that this exam has all 8 pages.

QUESTION	VALUE	SCORE
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL	100	

1. (20 points)

(a) Compute the area of surface of revolution obtained by rotating the curve $y = \sqrt{4 - x^2}$ around the x-axis.

ANSWER:

(b) Do the same for the curve $y = 1 - |x|$, $-1 \leq x \leq 1$.

ANSWER:

2. (20 points)

Consider the parametric curve

$x = \cos(t)$, $y = \sin(2t)$, $t \in [0, 2\pi]$.

(a)   At what points is the tangent horizontal or vertical?

ANSWER:

(b)   The curve passes through the origin twice. What are the slopes of the two tangent lines to the curve at the origin?

ANSWER:

(c)   Find the equation of the form $y = mx + b$ for the tangent at $t = \frac{\pi}{6}$.

ANSWER:

3. (20 points)

Find the arc-length of the parametric curve

$$x = 3\cos t - \cos 3t \,, \ y = 3\sin t - \sin 3t \,, \ 0 \le t \le \pi \,.$$

ANSWER:

4. (20 points)

(a)   Calculate the arc-length of the curve $r = \cos^2(\theta/2)$.

ANSWER:

(b)   Calculate the area enclosed by the curve $r^2 = \sin(2\theta)$.

ANSWER:

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5. (20 points)

(a) (5 points)   Does the sequence $\{a_n : n \geq 1\}$ with $a_n = 1/\sqrt{n}$ converge? Why or why not?

ANSWER:

(b) (5 points)   Use L'Hospital's Rule to show that for $k > 0$,

$$\lim_{x \to \infty} x^k \, e^{-x} = k \lim_{x \to \infty} x^{k-1} \, e^{-x} \, .$$

ANSWER:

c) (5 points)   Let $a_n = n^4\,e^{-n}$. Show that the sequence $\{a_n : n \geq 1\}$ converges. What is the limit?

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ANSWER:
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(d) (5 points)   Does the sequence $\{b_n : n \geq 1\}$ with $b_n = \sin(\frac{n\pi}{2})(-\frac{1}{3})^n$ converge? Why or why not?

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ANSWER:
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