

# Math 162: Calculus IIA

Second Midterm Exam

November 12, 2009

NAME (please print legibly): \_\_\_\_\_

Your University ID Number: \_\_\_\_\_

Indicate your instructor with a check in the box:

John Olsen	MWF 9:00 - 9:50 AM	<input type="checkbox"/>
Doug Ravenel	MWF 10:00 - 10:50 AM	<input type="checkbox"/>

- The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.
- Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- Put your answers in the boxes provided at the bottom of each page or half page. *You will not get credit for answers written elsewhere.*
- You are responsible for checking that this exam has all 8 pages.

QUESTION	VALUE	SCORE
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL	100	

**1. (20 points)**

What is the surface area of the surface of revolution obtained by rotating the infinite curve  $e^{-x}$ ,  $x \geq 0$  around the  $x$ -axis?

*You may use the formula*

$$\int \sec^3(x) dx = \frac{1}{2}(\sec(x) \tan(x) + \ln |\sec(x) + \tan(x)|) + C.$$

ANSWER:

**2. (20 points)**

Consider the parametric curve (an astroid or 4 pointed hypocycloid)  $x = \cos^3(t)$ ,  $y = \sin^3(t)$ ,  $t \in [0, 2\pi]$ .

(a) (7 points) At what points is the tangent horizontal or vertical?

ANSWER:

(b) (6 points) At what points does it have slope  $\pm 1$ ?

ANSWER:

(c) (7 points) Find the equation of the form  $y = mx + b$  for the tangent at  $t = \frac{\pi}{4}$ .

ANSWER:

**3. (20 points)**

Find the arc length of the cycloid  $x = r(t - \sin(t))$  and  $y = r(1 - \cos(t))$ , for  $0 \leq t \leq 2\pi$ .

ANSWER:

**4. (20 points)**

Consider the logarithmic spiral  $r = e^\theta$ ,  $\theta \geq 0$ , which can be defined parametrically by  $x = e^t \cos t$  and  $y = e^t \sin t$  with  $t = \theta$ .

(a) (10 points) Calculate the arc-length of the logarithmic spiral for  $0 \leq \theta \leq b$ .

ANSWER:

(b) (10 points) Calculate the area of the region between the  $x$ -axis and the curve for  $0 \leq \theta \leq \pi$ .

ANSWER:

**5. (20 points)**

(a) (5 points) Use L'Hospital's Rule to show that for  $k > 0$ ,

$$\lim_{x \rightarrow \infty} x^k e^{-x^2} = \frac{k}{2} \lim_{x \rightarrow \infty} x^{k-2} e^{-x^2} .$$

ANSWER:

(b) (5 points) Let  $a_n = n^8 e^{-n^2}$  where  $n = 1, 2, 3, \dots$ . Show that the sequence  $\{a_n : n \geq 1\}$  converges. What is the limit?

ANSWER:

(c) (5 points) Does the sequence  $b_n = \cos\left(\frac{n\pi}{2}\right)\left(-\frac{1}{2}\right)^n$  converge? Why or why not?

ANSWER:

(d) (5 points) Does the sequence  $b_n = \frac{1}{n^{0.005}}$  converge? Why or why not?

ANSWER: