Math 162: Calculus IIA

First Midterm Exam

September 26, 2024

NAME (please print legibly): _____

Your University ID Number: _____

Your University email _____

Indicate your instructor with a check in the box:

Nathanael Grand	MW 9:00 - 10:15 AM	
Doug Ravenel	MW 10:25 - 11:40 AM	
Peter Oberly	MW 12:30 - 1:45 PM	
Peter Oberly	MW 3:25 - 4:40 PM	

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam and that all work will be my own.

Signature: _____

- **The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden. IF YOU HAVE YOUR PHONE WITH YOU, YOU MUST TURN IT IN TO A PROCTOR BEFORE STARTING THE EXAM. FAILURE TO DO SO WILL BE TREATED AS AN ACADEMIC HONESTY VIOLATION.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **Put your answers in the space provided at the bottom of each page or half page.**
- **You are responsible for checking that this exam has all 11 pages.**

Integration by parts formula:

$$\int u\,dv = uv - \int v\,du$$

Trigonometric identities:

$$\cos^2(x) + \sin^2(x) = 1 \qquad \sec^2(x) - \tan^2(x) = 1 \qquad \sin(2x) = 2\sin(x)\cos(x)$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2} \qquad \sin^2(x) = \frac{1-\cos(2x)}{2}$$

Derivatives of trig functions.

$$\frac{d\sin x}{dx} = \cos x \qquad \frac{d\tan x}{dx} = \sec^2 x \qquad \frac{d\sec x}{dx} = \sec x \tan x$$

$$\frac{d\cos x}{dx} = -\sin x \qquad \frac{d\cot x}{dx} = -\csc^2 x \qquad \frac{d\csc x}{dx} = -\csc x \cot x$$

Trigonometric substitution tricks for odd powers of secant and even powers of tangent:

$$u = \sec(\theta) + \tan(\theta) \qquad \sec(\theta)d\theta = \frac{du}{u}$$

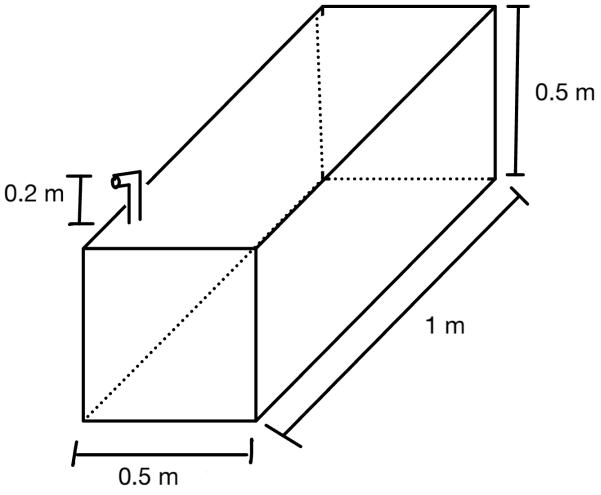
$$\sec(\theta) = \frac{u^2+1}{2u} \qquad \tan(\theta) = \frac{u^2-1}{2u}$$

1. (20 points) Let $c > 0$ be a fixed number. Find the area between the two curves $x = cy$ and $x = cy^2$. Note that your answer will depend on c.

ANSWER:

2. (20 points) This is a work problem in metric units. A fish tank in the shape of a rectangular prism has dimensions 0.5 m $\times$ 0.5 m $\times$ 1 m, as pictured below.

There is a spout on top of the fish tank whose outlet is 0.2 meters above the top of the tank. Suppose the tank is filled with water to a depth of 0.25 meters. How much work does it take to pump all of the water out through the spout? Use $\rho = 1,000$ kilograms per meter cubed for the density of water, and use $g = 10$ meters per second squared for the acceleration due to gravity.



ANSWER:

3. (20 points) (a) (15 points) Let $n \geq 2$ be an integer. Find numbers A_1, A_2, and A_3 so that

$$\int x^n \cos(x)\, dx = A_1 x^n \sin(x) + A_2 x^{n-1} \cos(x) + A_3 \int x^{n-2} \cos(x)\, dx.$$

[Hint: Integrate $x^n \cos(x)$ by parts twice. Note that some of the numbers A_1, A_2, and A_3 could depend on n.]

ANSWER:

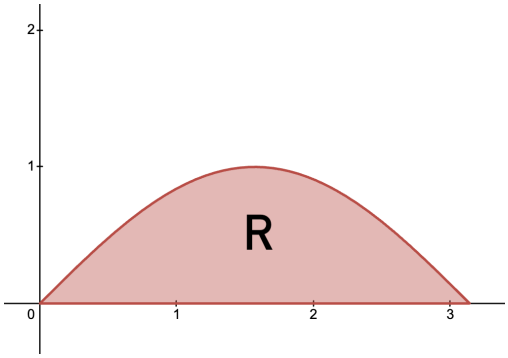
(b) (5 points) Evaluate

$$\int_0^{\pi} x^2 \cos(x)\, dx.$$

Note: If you apply the formula from part (a) and your values for A_1, A_2, and A_3 are incorrect, you may not receive credit for this problem.

ANSWER:

4. (20 points) Let R be the space between the graph of $f(x) = \sin(x)$ and the x-axis, which lies in between the vertical lines $x = 0$ and $x = \pi$. Find the volume of the solid generated when rotating the region R about the horizontal line $y = 2$.



ANSWER:

5. (20 points) Evaluate

$$\int \cos^5(2x+1)\,dx.$$

ANSWER:

Scratch paper

Scratch paper

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