Math 162: Calculus IIA

First Midterm Exam

October 10, 2023

NAME (please print legibly): _____

Your University ID Number: _____

Your University email _____

Indicate your instructor with a check in the box:

Firdavs Rakhmonov	MW 9:00 - 10:15 AM	
Doug Ravenel	MW 10:25 - 11:40 AM	
Peter Oberly	MW 12:30 - 1:45 PM	
Sefika Kuzgun	MW 3:25 - 4:40 PM	

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam and that all work will be my own.

Signature: _____

- **The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden. IF YOU HAVE YOUR PHONE WITH YOU, YOU MUST TURN IT IN TO A PROCTOR BEFORE STARTING THE EXAM. FAILURE TO DO SO WILL BE TREATED AS AN ACADEMIC HONESTY VIOLATION.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **Put your answers in the space provided at the bottom of each page or half page.**
- **You are responsible for checking that this exam has all 14 pages.**

Integration by parts formula:

$$\int u\,dv = uv - \int v\,du$$

Trigonometric identities:

$$\cos^2(x) + \sin^2(x) = 1 \qquad \sec^2(x) - \tan^2(x) = 1$$
$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos^2(x) = \frac{1+\cos(2x)}{2} \qquad \sin^2(x) = \frac{1-\cos(2x)}{2}$$

Derivatives of trig functions.

$$\frac{d\sin x}{dx} = \cos x \qquad \frac{d\tan x}{dx} = \sec^2 x \qquad \frac{d\sec x}{dx} = \sec x \tan x$$
$$\frac{d\cos x}{dx} = -\sin x \qquad \frac{d\cot x}{dx} = -\csc^2 x \qquad \frac{d\csc x}{dx} = -\csc x \cot x$$

Trigonometric substitution (known in Doug's section as *the rabbit trick.*)) for odd powers of secant and even powers of tangent:

$$u = \sec(\theta) + \tan(\theta) \qquad \sec(\theta)d\theta = \frac{du}{u}$$
$$\sec(\theta) = \frac{u^2+1}{2u} \qquad \tan(\theta) = \frac{u^2-1}{2u}$$

1. (20 points)

(a) (10 points) Find the integral

$$\int \frac{x}{\sqrt{x^2 - 6x + 13}}\, dx$$

ANSWER:

(b) (10 points) Find the integral

$$\int \frac{2x+1}{x^3+2x^2+x}\,dx.$$

ANSWER:

2. (20 points)

(a) (10 points) Prove the reduction formula

$$\int \sin^n x\, dx = -\frac{1}{n}\cos x \sin^{n-1} x + \frac{n-1}{n}\int \sin^{n-2} x\, dx,$$

where $n \geq 2$ is an integer.

ANSWER:

(b) (10 points) Use your formula repeatedly to find

$$\int \sin^4 x \, dx$$

ANSWER:

3. (20 points) If $a \neq 0$, evaluate

$$\int \sin^2(ax)\cos^2(ax)\,dx$$

in terms of a.

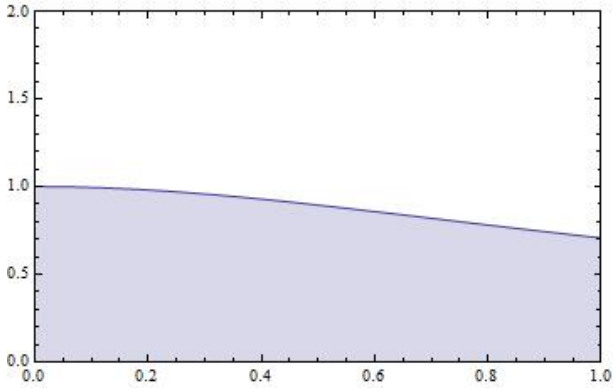
ANSWER:

4. (20 points) This a work problem with metric units. Assume that acceleration due to gravity is A meters per second per second. You should give your answer in joules as a multiple of $A\pi$. The density of water is a thousand kilograms per cubic meter.

Consider the region of the xy-plane bounded by the curve $y = x^2$ and the lines defined by $x = 0$ and $y = 3$. Rotate this region about the y-axis to obtain a solid region or bowl, which is filled with water. How much work is needed to pump the water about over the top of the bowl?

ANSWER:

5. (20 points)



Consider the region $\mathcal{R}$ bounded by the x-axis, y-axis, the line $x = 1$, and $y = \frac{1}{\sqrt{1+x^2}}$.

(a) **(10 points)** Compute the volume of the solid obtained by revolving $\mathcal{R}$ about the x-axis.

ANSWER:

(b) **(10 points)** Compute the volume of the solid obtained by revolving $\mathcal{R}$ about the y-axis.

ANSWER:

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Scratch paper

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