

Math 162: Calculus IIA

First Midterm Exam

October 13, 2022

NAME (please print legibly): _____

Your University ID Number: _____

Your University email _____

Indicate your instructor with a check in the box:

| | | |
|---------------|----------------------|--------------------------|
| Sefika Kuzgun | MW 9:00 - 10:15 AM | <input type="checkbox"/> |
| Doug Ravenel | MWF 10:25 - 11:40 AM | <input type="checkbox"/> |
| Josh Sumpter | TR 9:40 - 10:55 AM | <input type="checkbox"/> |
| Carissa Slone | TR 2:00 - 3:15 PM | <input type="checkbox"/> |

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam and that all work will be my own.

Signature: _____

- The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden. **IF YOU HAVE YOUR PHONE WITH YOU, YOU MUST TURN IT IN TO A PROCTOR BEFORE STARTING THE EXAM. FAILURE TO DO SO WILL BE TREATED AS AN ACADEMIC HONESTY VIOLATION.**
- Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- Put your answers in the space provided at the bottom of each page or half page.
- You are responsible for checking that this exam has all 16 pages.

Integration by parts formula:

$$\int u dv = uv - \int v du$$

Trigonometric identities:

$$\cos^2(x) + \sin^2(x) = 1$$

$$\sec^2(x) - \tan^2(x) = 1$$

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\cos^2(x) = \frac{1 + \cos(2x)}{2}$$

$$\sin^2(x) = \frac{1 - \cos(2x)}{2}$$

Derivatives of trig functions.

$$\frac{d \sin x}{dx} = \cos x$$

$$\frac{d \tan x}{dx} = \sec^2 x$$

$$\frac{d \sec x}{dx} = \sec x \tan x$$

$$\frac{d \cos x}{dx} = -\sin x$$

$$\frac{d \cot x}{dx} = -\csc^2 x$$

$$\frac{d \csc x}{dx} = -\csc x \cot x$$

Trigonometric substitution tricks for odd powers of secant and even powers of tangent:

$$u = \sec(\theta) + \tan(\theta)$$

$$\sec(\theta)d\theta = \frac{du}{u}$$

$$\sec(\theta) = \frac{u^2 + 1}{2u}$$

$$\tan(\theta) = \frac{u^2 - 1}{2u}$$

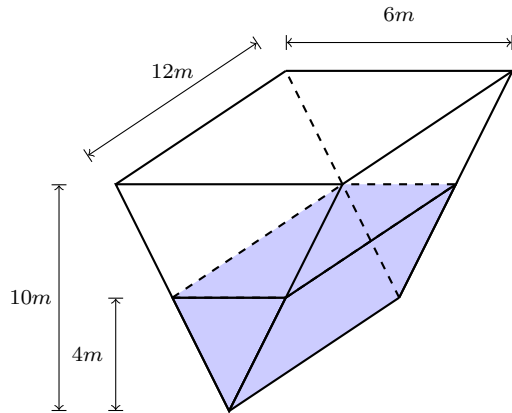
1. (20 points) If $a \neq 0$, evaluate

$$\int \cos^3(ax + b) dx$$

in terms of a and b .

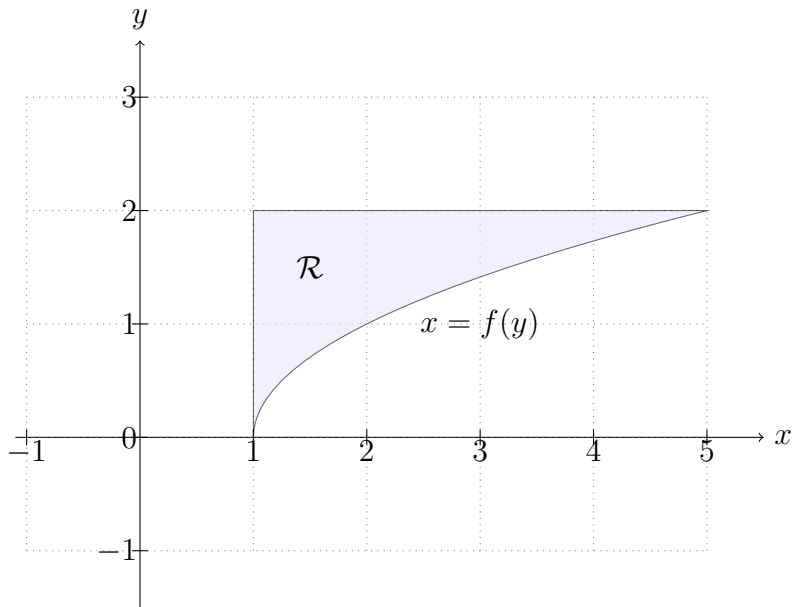
ANSWER:

2. (20 points) Consider a tank that is 10 meters tall with sides in the shapes of congruent isosceles triangles and a rectangular top that is 6 meters wide and 12 meters in length (see diagram below). The tank is filled with water to a depth of 4 meters. Find the work done pumping the water to a point 1 meter above the top of the tank. (The water density $\rho = 1000kg/m^3$ and the gravity constant is $g = 10m/s^2$). You do not need to simplify your answer.



ANSWER:

3. (20 points) Set up formulas using integral expressions for the volumes of the following solids related to the region \mathcal{R} where integration is performed with respect to the variable y .



(a) (4 points) The solid resulting from rotating \mathcal{R} about the x -axis.

ANSWER:

(b) (4 points) The solid resulting from rotating \mathcal{R} about the y -axis.

ANSWER:

(c) (4 points) The solid resulting from rotating \mathcal{R} about the axis $x = 5$.

ANSWER:

(d) (4 points) The solid resulting from rotating \mathcal{R} about the axis $y = 3$.

ANSWER:

(e) (4 points) The solid with base \mathcal{R} where cross-sections parallel to the x -axis are squares.

ANSWER:

4. (20 points)

(a) (10 points) Use integration by parts to find a formula for

$$\int x^n e^x dx \quad \text{in terms of} \quad \int x^{n-1} e^x dx$$

for any integer $n \geq 0$.

ANSWER:

(b) (10 points) Use your formula repeatedly to find

$$\int x^3 e^x dx$$

ANSWER:

5. (20 points) (a) (10 points) Find the integral

$$\int \sqrt{x^2 - 8x + 17} dx$$

ANSWER:

(b) (10 points) Find the integral

$$\int \frac{x+1}{x^3+x} dx.$$

ANSWER:

Scratch paper

Scratch paper

Scratch paper