

# Math 162: Calculus IIA

First Midterm Exam

October 20, 2009

NAME (please print legibly): \_\_\_\_\_

Your University ID Number: \_\_\_\_\_

Indicate your instructor with a check in the box:

John Olsen	MWF 9:00 - 9:50 AM	<input type="checkbox"/>
Doug Ravenel	MWF 10:00 - 10:50 AM	<input type="checkbox"/>

- The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.
- Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- Put your answers in the space provided at the bottom of each page or half page.
- You are responsible for checking that this exam has all 11 pages.

QUESTION	VALUE	SCORE
1	20	
2	20	
3	20	
4	20	
5	20	
TOTAL	100	

**1. (20 points)**

(a) (10 points) Find a partial fraction expansion for the function

$$\frac{1}{x^3 - x^2 + 2x - 2}.$$

ANSWER:

1. (b) (10 points) Calculate the integral

$$\int \frac{dx}{x^3 - x^2 + 2x - 2}.$$

ANSWER:

**2. (20 points)** Consider the curve  $y = x^{3/2}$

(a) (10 points) Calculate the arc length function starting at  $x = 0$ .

ANSWER:

2. (b) (10 points) Calculate the arc length from  $x = 4$  to  $x = 8$ .

ANSWER:

**Solution:** (a)  $y' = \frac{3}{2}\sqrt{x}$ , so by substituting  $u = 1 + \frac{9}{4}x$  one gets

$$\begin{aligned} s(t) &= \int_0^t \sqrt{\left(1 + \frac{9}{4}x\right)} dx \\ &= \frac{4}{9} \int_1^{1+9t/4} \sqrt{u} du \\ &= \frac{8}{27} u^{3/2} \Big|_1^{1+9t/4} \\ &= \frac{8}{27} \left(1 + \frac{9}{4}x\right)^{3/2} - \frac{8}{27} \end{aligned}$$

for  $t \geq 0$ .

(b) By the definition of the arc length function,  $s(4)$  is the arclength from  $t = 0$  to  $t = 4$  and  $s(8)$  is the arclength from  $t = 0$  to  $t = 8$ , so the arc length from  $t = 4$  to  $t = 8$  is

$$s(8) - s(4) = \frac{8}{27} (19^{3/2} - 10^{3/2}) = \frac{8}{27} (19\sqrt{19} - 10\sqrt{10}).$$

**3. (20 points)** Consider region between the curve  $y = \sin^2 x$  for  $0 \leq x \leq \pi$  and the  $x$ -axis.

(a) Find the volume of the solid of revolution about the  $x$ -axis.

ANSWER:
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3. (b) Find the volume of the solid of revolution about the  $y$ -axis.

ANSWER:

**4. (20 points)**

(a) (10 points) Use integration by parts to find a formula for

$$\int x^n e^x dx \quad \text{in terms of} \quad \int x^{n-1} e^x dx$$

ANSWER:



(b) (10 points) Use this formula to find

$$\int x^3 e^x dx.$$

ANSWER:

5. (20 points) Consider the integral

$$\int \frac{dx}{\sqrt{4x^2 - 12x}}$$

(a) (5 points) Write the quantity under the square root sign as a sum or difference of two squares.

ANSWER:

(b) (5 points) Draw a right triangle in which one of the sides is the square root in the integral and another is a constant.

ANSWER:

5. (c) (10 points) Evaluate

$$\int_3^4 \frac{dx}{\sqrt{4x^2 - 12x}}.$$

ANSWER: