

Math 162

First Midterm

October 24, 2006

NAME (please print legibly): _____

Your University ID Number: _____

Circle your Instructor's Name along with the Lecture Time:

Brigitta Vermesi (9:00-10:00) Carl Mueller (10:00-11:00)

- No calculators are allowed on this exam.
- Please show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.
- Please put your simplified final answers in the spaces provided.

QUESTION	VALUE	SCORE
1	8	
2	8	
3	8	
4	10	
5	10	
6	10	
7	10	
8	10	
9	8	
10	8	
11	10	
TOTAL	100	

Here are the double angle formulas:

$$\sin x \cos x = \frac{1}{2} \sin(2x)$$

$$\cos^2 x = \frac{1}{2} (1 + \cos(2x))$$

$$\sin^2 x = \frac{1}{2} (1 - \cos(2x))$$

Some more trig formulas:

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\sec^2 x - 1 = \tan^2 x$$

1. (8 points)

Solve the integral

$$\int x^3 \sqrt{x^2 + 3} dx$$

2. (8 points)

Solve the integral

$$\int \frac{1}{(9 - 4x^2)^{3/2}} dx$$

3. (8 points)

Solve the integral

$$\int \frac{x^2 + 3x}{x^2 + 3x + 2} dx$$

4. (10 points)

The density of water is 1000 kg/m^3 and the gravitational constant is 9.8 m/s^2 .

A tank has a parabolic shape, obtained by rotating the curve $y = x^2$ about the y -axis, between $y = 0$ and $y = 16$. Distance along both axes is measured in meters. The tank is filled with water to the top. How much work does it take to bring all the water to the top of the tank?

5. (10 points)

Suppose that the region bounded by

$$y = 0$$

$$y = \sin x$$

$$x = 0$$

$$x = \pi$$

is rotated about the y -axis. Find the volume of the resulting region.

6. (10 points)

It takes 10 ft-lb of work to stretch a spring from its rest position of 3 ft to 5 ft. How much work would it take to stretch it from 5 ft to 8 ft?

7. (10 points)

Find the area of the finite region bounded by the curves $y = x^2$ and $y = x + 2$.

8. (10 points)

Find the volume of the solid obtained by rotating the region bounded by the curves $x = y^2$, $x = 4$ and $y = 0$ about the x -axis.

9. (8 points)

Evaluate the indefinite integral

$$\int e^{\sqrt{x}} dx$$

10. (8 points)

Solve this integral

$$\int_0^{\pi/2} \sin^3 \theta \cos^2 \theta d\theta$$

11. (10 points)

Determine whether the integral

$$\int_1^{\infty} \frac{\ln x}{x^2} dx$$

is divergent or convergent. For full credit, be sure to explain your reasoning. If it is convergent, evaluate it. If not, state your answer as “divergent.”