MATH 162: Calculus IIA

Practice for Midterm 1

Februrary 22, 2007

8:00 AM in Hutchison 140 & 141

(Lander and Hubbell Auditoriums)

1. Evaluate the following integrals. State the method you use to find the antiderivatives. (Make up some limits of integration to practice definite integrals, especially when substitution or integration by parts is used.)

$$\int \frac{x}{\sqrt{1-8x^2}}\,dx \qquad \int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx$$

$$\int \sin\sqrt{x}\,dx \qquad \int (\ln(x))^2\,dx$$

$$\int \sin^2 t\cos^2 t\,dt \qquad \int \frac{1}{(4+x^2)^{3/2}}\,dx$$

$$\int \tan^5 t\sec^4 t\,dt \qquad \int \sqrt{5+4x-x^2}\,dx$$

$$\int \cos^2(x)\sin(2x)\,dx \qquad \int x\ln(x)\,dx$$

$$\int e^{-x}\cos(2x)\,dx \qquad \int x^7\sqrt{1+x^2}\,dx$$

2. A tank has the shape of an inverted circular cone with height 8 m and square base of radius 4 m. The tank is full of water. Find the work required to empty the tank by pumping all of the water to the top of the tank. (Use the fact that water density is 1000 kg per cubic meter.)

3. A swimming pool has the shape of a cylinder with (horizontal) radius 5 meters and depth 3 meters. Assume that the pool contains water to a depth of 2 meters. Find the work required to pump all of the water out over the top of the pool. (The acceleration of gravity is $g = 9.8$ meters per second squared.)

4. If 6 J of work is needed to stretch a spring from 10 cm to 12 cm and another 10 J is needed to stretch it from 12 cm to 14 cm, what is the natural length of the spring? (Assume the spring lies along the x-axis with its base at $x = 0$.)

5. Find the area of the region enclosed by the given curves:

   (a) $y = 6 - x^2$ and $y = x^2 - 2$

   (b) $x = y^2 - 2$, $x = e^y$, $y = 1$ and $y = -1$.

(c) $y = 4x + 3$ and $y = x^3 + 3$.

6. What is the volume of the solid obtained by rotating the region bounded by $y = 1/x$, $y = 0$, $x = 1$ and $x = 3$ about the line $y = -1$.

7. Using the method of cylindrical shells, find the volume of the solid obtained by rotating the region bounded by the x-axis and $y = \sin(x^2)$ about the y-axis.

8. Using the method of cylindrical shells, find the volume of the solid obtained by rotating the region bounded by the x-axis, $x + y = 3$ and $x = 4 - (y - 1)^2$ about the y-axis.