Math 162: Calculus IIA

First Midterm Exam

September 30, 2021

NAME (please print legibly): _____

Your University ID Number: _____

Your University email _____

Indicate your instructor with a check in the box:

Bogdan Krstic	MW 9:00 - 10:15 AM	
Doug Ravenel	MWF 10:25 - 11:40 AM	
Charles Wolf	MW 12:30 - 1:45 PM	

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam and that all work will be my own.

Signature: _____

- **The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden. IF YOU HAVE YOUR PHONE WITH YOU, YOU MUST TURN IT IN TO A PROCTOR BEFORE STARTING THE EXAM. FAILURE TO DO SO WILL BE TREATED AS AN ACADEMIC HONESTY VIOLATION.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given. If some of your work is not on the page where the problem appears, indicate where it is.**
- **Put your answers in the space provided at the bottom of each page or half page.**
- **You are responsible for checking that this exam has all 9 pages.**

Integration by parts formula:

$$\int u\,dv = uv - \int v\,du$$

Trigonometric identities:

$$\cos^2(x) + \sin^2(x) = 1 \qquad \sec^2(x) - \tan^2(x) = 1 \qquad \sin(2x) = 2\sin(x)\cos(x)$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2} \qquad \sin^2(x) = \frac{1-\cos(2x)}{2}$$

Derivatives of trig functions.

$$\frac{d\sin x}{dx} = \cos x \qquad \frac{d\tan x}{dx} = \sec^2 x \qquad \frac{d\sec x}{dx} = \sec x\tan x$$

$$\frac{d\cos x}{dx} = -\sin x \qquad \frac{d\cot x}{dx} = -\csc^2 x \qquad \frac{d\csc x}{dx} = -\csc x\cot x$$

1. (20 points)

Compute the following integral:

$$\int \cos^{2021}(x) \tan^3(x) dx$$

ANSWER:

2. (20 points)

The average value of a function $f(x)$ for $a \leq x \leq b$ is

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

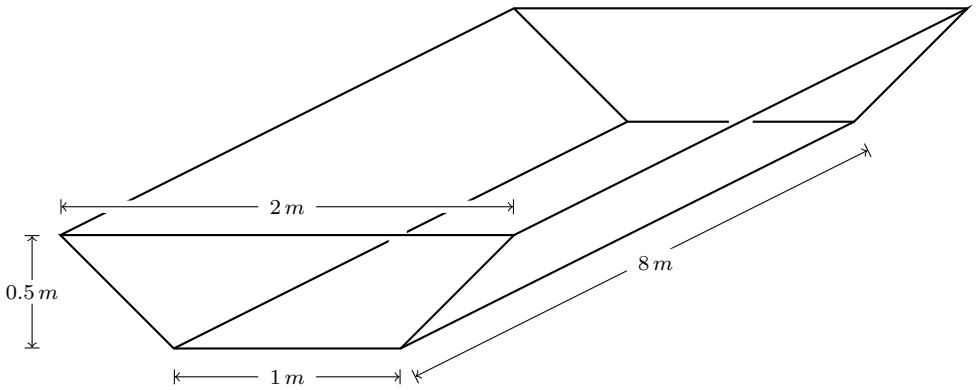
Find the average value of the function $f(x) = 2 + \sin x$ for $0 \leq x \leq 5\pi$. HINT: You may use the fact that

$$\int_0^{\pi} \sin x\,dx = 2 \qquad \text{and} \qquad \int_a^{a+2\pi} \sin x\,dx = 0 \qquad \text{for any number } a.$$

ANSWER:

3. (20 points)

A trough is 4 meters long and half a meter tall, with vertical cross-sections parallel to the ends in the shape of isosceles trapezoids which are 1 meter wide at the bottom and 2 meters wide at the top. The trough is full of water. Find work done pumping the water to the top of the trough. Assume that the water density is $\rho = 1000\,kg/m^3$ and the gravity constant is $g = 10\,m/s^2$.



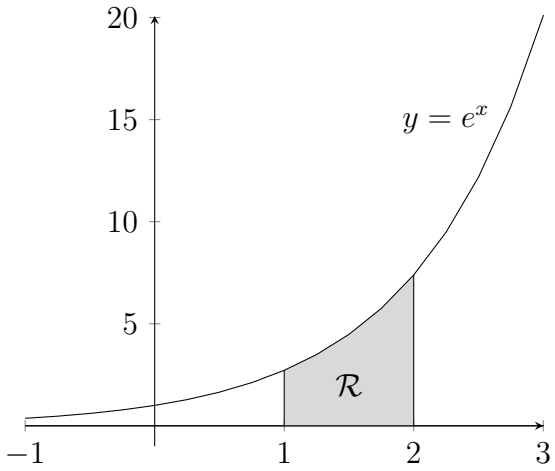
ANSWER:

4. (20 points)

The region $\mathcal{R}$ in the plane is bounded by the graph of $y = e^x$, the x-axis, and the lines $x = 1$ and $x = 2$.

(a) Find the volume of the solid $\mathcal{S}$ obtained by revolving $\mathcal{R}$ about the x-axis.

(b) Find the volume of the solid $\mathcal{T}$ obtained by revolving $\mathcal{R}$ about the y-axis.



ANSWER:

5. (20 points)

(a) Use integration by parts to find a formula for

$$\int x^n e^x \, dx \qquad \text{in terms of} \qquad \int x^{n-1} e^x \, dx$$

for any integer $n \geq 0$.

(b) Use your formula repeatedly to find

$$\int x^3 e^x \, dx$$

ANSWER:

Scratch paper

More scratch paper