

HANDY DANDY FORMULAS

Integration by parts formula: $\int u dv = uv - \int v du$

Trigonometric identities:

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

Derivatives of trig functions.

$$\frac{d \sin x}{dx} = \cos x$$

$$\frac{d \tan x}{dx} = \sec^2 x$$

$$\frac{d \sec x}{dx} = \sec x \tan x$$

$$\frac{d \cos x}{dx} = -\sin x$$

$$\frac{d \cot x}{dx} = -\csc^2 x$$

$$\frac{d \csc x}{dx} = -\csc x \cot x$$

Area of surface of revolution in rectangular coordinates for $y = f(x)$ with $a \leq x \leq b$

- about the x -axis: $S = 2\pi \int_a^b f(x) \sqrt{1 + f'(x)^2} dx$

- about the y -axis: $S = 2\pi \int_a^b x \sqrt{1 + f'(x)^2} dx$

Polar coordinates

$$r = \sqrt{x^2 + y^2} \quad \theta = \begin{cases} \arctan(y/x) & \text{for } x > 0 \\ \pi + \arctan(y/x) & \text{for } x < 0 \\ \pi/2 & \text{for } x = 0 \text{ and } y > 0 \\ 3\pi/2 & \text{for } x = 0 \text{ and } y < 0 \\ \text{undefined} & \text{for } (x, y) = (0, 0) \end{cases}$$

$$x = r \cos \theta, y = r \sin \theta$$

Changing θ by any multiple of 2π does not change the location of the point. Changing the sign of r is equivalent to adding π to θ , which is the same as moving the point to one in the opposite direction and the same distance from the origin.



Area in polar coordinates for $r = f(\theta)$ with $\alpha \leq \theta \leq \beta$:

$$A = \int_{\alpha}^{\beta} \frac{r^2}{2} d\theta$$

Arc length formulas

- Rectangular coordinates, $y = f(x)$ with $a \leq x \leq b$:

$$S = \int_a^b \sqrt{1 + f'(x)^2} dx$$

- Polar coordinates, $r = f(\theta)$ with $\alpha \leq \theta \leq \beta$:

$$S = \int_{\alpha}^{\beta} \sqrt{r^2 + f'(\theta)^2} d\theta$$

- Parametric equations, $x = x(t)$ and $y = y(t)$ with $a \leq t \leq b$:

$$S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

Infinite series formulas

- The *Maclaurin series* for $f(x)$ is $\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$.
- The *Taylor series* for $f(x)$ at a is $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n$.
- The *mth Taylor polynomial* is $T_m(x) = \sum_{n=0}^m \frac{f^{(n)}(a)}{n!} (x - a)^n$,
- The *mth Taylor remainder* is $R_m(x) = f(x) - T_m(x)$
- Taylor's inequality says that if $|f^{(n+1)}(x)| \leq M$ for suitable x , then $|R_m(x)| \leq \frac{(x - a)^{n+1} M}{(n + 1)!}$.

