# Math 162: Calculus IIA

Final Exam

May 6, 2019

NAME (please print legibly): _____

Your University ID Number: _____

Your University email _____

Indicate your instructor with a check in the box:

| Saul Lubkin | MW 9:00 - 10:15 AM | |
| Doug Ravenel | MWF 10:25 - 11:40 AM | |
| Rufei Ren | MW 2:00 - 3:15 PM | |
| Martin Snow | MW 3:25 - 4:40 PM | |
| Amanda Tucker | TR 9:40-10:55 AM | |

## Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam and that all work will be my own.

Signature: _____

- The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden and **WILL BE TREATED AS AN ACADEMIC HONESTY VIOLATION.**

- Show your work and justify your answers. Put your answers in the space provided at the bottom of each page or half page. **SIMPLIFY YOUR ANSWERS AS MUCH AS POSSIBLE.**

- You are responsible for checking that this exam has all 14 pages.

- Part A (problems 1–6) covers the same material as the two midterms, and Part B (problems 7–11) covers additional material. Letter grades will be computed for the two parts separately. Part B will count for 20% of your course grade. Part A will count for at least 10% of your course grade. If your letter grade on part A is better than your lowest midterm letter exam grade, then it will replace that midterm exam grade and count for 30% of your course grade.

## Part A

**1. (20 points)**

Find the arc length $L$ of the parametric curve, $x = 2t$, $y = 4\ln((t/2)^2 - 1)$, from $t = 6$ to $t = 7$.
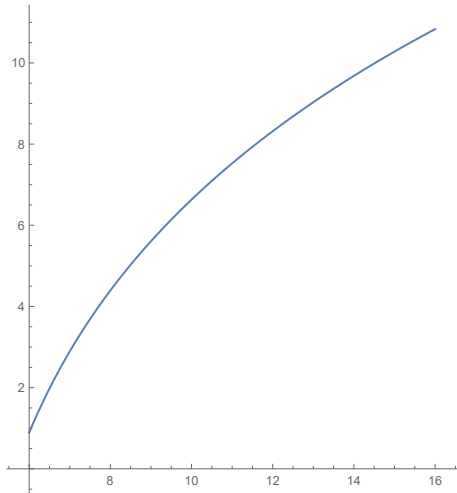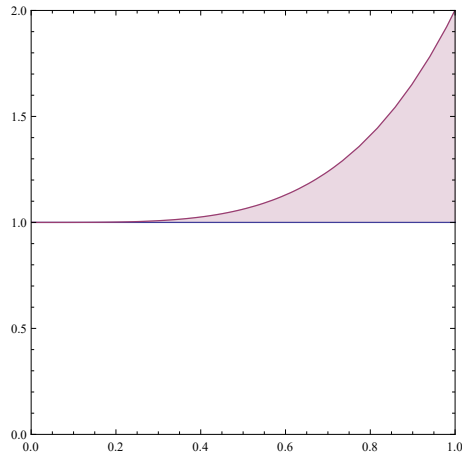


ANSWER:

**2. (20 points)** Compute

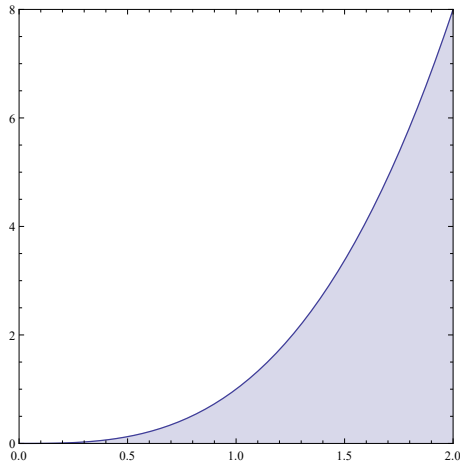$$\int \frac{1}{\sqrt{1 + (6x - 4)^2)}}\,dx$$

ANSWER:

### 3. (20 points)

(a) Compute the volume of a region bounded by the curves $y = x^4 + 1$, $y = 1$ and $x = 1$ and rotated around the $y$-axis.



ANSWER:

(b) Set up the integral for the volume of the region bounded by $y = x^3$, $y = 0$ and $x = 2$ and rotated around line $x = 2$. Use the shell method. Do not evaluate the integral.



ANSWER:

**4. (10 points)**

Evaluate the integral

$$\int \arctan(2x)dx.$$

ANSWER:

**5. (20 points)**

(a) Find the partial fraction decomposition of

$$\frac{x^2 + 3x}{x^2 - 4}.$$

ANSWER:

(b) Write out the form of the partial fraction decomposition of the function

$$\frac{x^3 - 5}{(x+1)^3(x^2+4)^2(x-1)} = \underline{\hspace{6cm}}$$

Do not determine the numerical values of the coefficients.

ANSWER:
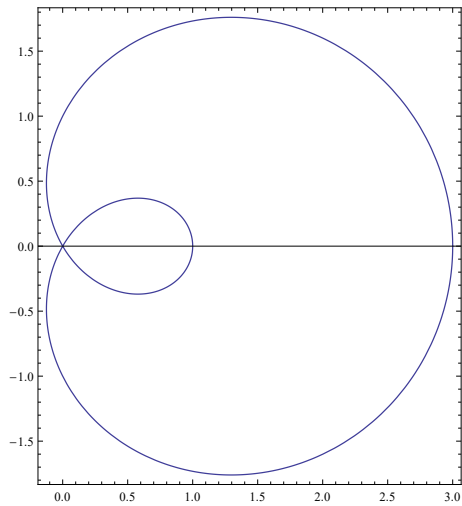
(c) Let

$$f(x) = \frac{1}{x-1} + \frac{2x+3}{x^2+1}.$$

Evaluate

$$\int f(x)dx.$$

ANSWER:

**6. (15 points)**

Find the area inside the outer (larger) loop but outside the inner (smaller) loop of the limaçon
$r = 1 + 2\cos(\theta)$.



ANSWER:

## Part B

**7. (20 points)**

(a) Determine whether the series

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^6}$$

is absolutely convergent, conditionally convergent, or divergent.

ANSWER:

(b) Estimate the sum of the series with an accuracy of $.01 = 1/100$.

ANSWER:

**8. (20 points)**

(a) Find a power series representation centered at $-1$ as well as the radius and interval of convergence for the function

$$f(x) = \frac{x+1}{x-1}$$

ANSWER:

(b) Write the following integral as a power series in $x + 1$. What is the radius of convergence of this power series?

$$\int \frac{x+1}{x-1} dx$$

ANSWER:

**9. (20 points)**

Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n - \ln n}$$

ANSWER:

**10. (20 points)**

Find the radius of convergence and interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{2^n (x - 3)^n}{\sqrt{n}}.$$

ANSWER:

**11. (20 points)** Let $f(x) = \dfrac{x}{x^2 + 4}$.

(a) Find a power series expansion for $f(x)$ about $x = 0$. Write it in the form $\displaystyle\sum_{n=0}^{\infty}(-1)^{e_n} a_n x^{p_n}$.

ANSWER:

(b) Find the radius and interval of convergence for the series you found in (a).

ANSWER:

(c) Find $f^{(5)}(0)$ and $f^{(10)}(0)$.

ANSWER:

Scratch paper

More scratch paper

And even more scratch paper