Math 162: Calculus IIA

Final Exam

May 2nd, 2016

Please circle your section:

Gage MW 2:00pm **Harper TR 9:40am** **Lubkin MWF 9:00am**

Lungstrum MW 3:25pm **Neuman TR 4:50pm** **Tucker MWF 10:25am**

NAME (please print legibly): ____________________

Your University ID Number: ____________________

Your University E-mail: ____________________

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam and that all work will be my own.

Signature: ____________________

Part A		
QUESTION	VALUE	SCORE
1	10	
2	10	
3	15	
4	15	
5	15	
6	15	
7	20	
TOTAL	100	

Part B		
QUESTION	VALUE	SCORE
8	15	
9	15	
10	10	
11	15	
12	10	
13	15	
14	10	
15	10	
TOTAL	100	

Instructions:

- **The use of calculators, cell phones, iPods, and other electronic devices at this exam is strictly forbidden. You must be physically separated from your cell phone.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **Put your answers in the spaces provided.**
- **You are responsible for checking that this exam has all 17 pages.**

Formulas:

- $\sin(x)\cos(x) = \frac{1}{2}\sin(2x)$
- $\sin^2(\theta) + \cos^2(\theta) = 1$
- $\tan^2(\theta) + 1 = \sec^2(\theta)$
- $\cos^2(\theta) = \frac{1}{2}(1 + \cos(2\theta))$
- $\sin^2(\theta) = \frac{1}{2}(1 - \cos(2\theta))$
- $\int \tan(x)dx = \ln|\sec(x)| + C$
- $\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + C$
- $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$
- $\int \sec^3(x)dx = \sec(x)\tan(x) + \ln|\sec(x) + \tan(x)| + C$

Part A

1. (10 points) The region between the x-axis and the curve $y = -2x$ for $0 \leq x \leq 2$ is rotated about the line $y = 1$. Compute the volume.

2. (10 points) The region between the x-axis and the curve $y = \sin(x^2)$ for $0 \leq x \leq \sqrt{\pi}$ is rotated about the y-axis. Compute the volume.

3. (15 points)

(a) Compute the indefinite integral $\displaystyle\int \tan^3(x)\sec^4(x)\,dx$.

(b) Find the average value of the function $f(x) = \tan^3(x)\sec^4(x)$ on the interval $[0, \frac{\pi}{3}]$.

4. (15 points)

(a) Find the following indefinite integral.

$$\int \frac{dx}{x^2 - 8x + 25}$$

(b) Set up the partial fraction decomposition for the following integral in terms of variables, but do not solve for those variables.

$$\int \frac{x^2 - 8x + 25}{x(x^3 + x)}\, dx$$

5. (15 points) Consider the parametric curve defined by

$$x(t) = t^2 + 2$$
$$y(t) = t^3 - t$$

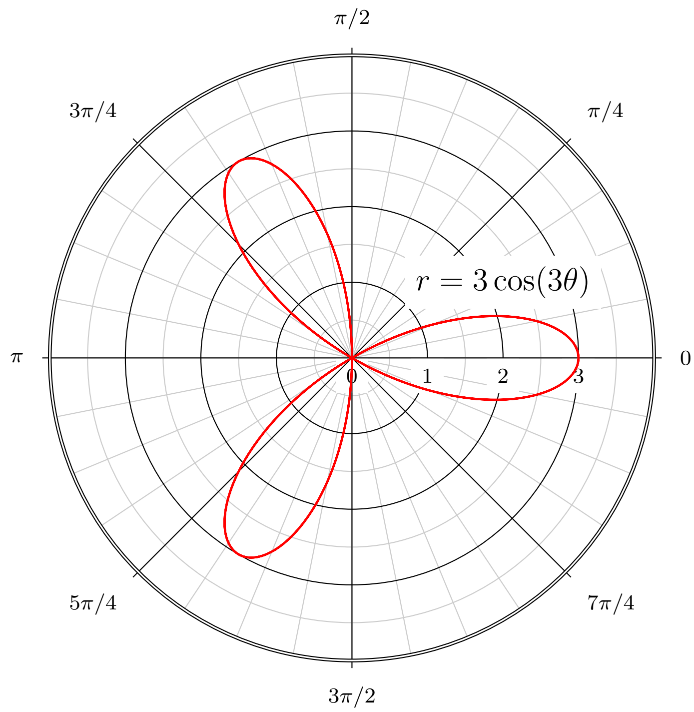
(a) Find two different tangent lines to the curve at the point $(3, 0)$.

(b) For which values of t does the curve have a vertical tangent line?

(c) Set up, but do **NOT** evaluate, an integral to calculate the arc length of the curve from $t = 0$ to $t = 4$.

6. (15 points)

Use the polar area formula to find the area of one leaf of the three-leafed rose $r = 3\cos 3\theta$.



7. (20 points) Determine whether each of the following sequences converge. If a sequence converges find its limit; if it diverges, explain why.

(a) $a_n = \dfrac{2n}{3n+1}$

(b) $a_n = \ln(2n) - \ln(3n+1)$

(c) $a_n = \dfrac{n}{\ln(n+1)}$

(d) $a_n = \dfrac{\sin(n)}{\sqrt{n}}$

(e) $a_n = \sin\left(\frac{1}{\sqrt{n}}\right)$

Part B

8. (15 points) Determine whether each of the following series diverges or converges. If a series diverges, justify. If it converges, justify, and find the value of its sum.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{2n}{3n+1}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n + (-2)^n}{3^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{n(n+2)}$

9. (15 points)

(a) Use the Integral Test to check whether the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)}$ converges or diverges.

(b) Determine all values of p for which the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$$

converges.

10. (10 points) Determine if the following series converge or diverge and justify your answer.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n}{3n+1}\right)^n$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\arctan(n)}{n^{1.2}}$

11. (15 points) Decide whether the following series are absolutely convergent, conditionally convergent, or divergent, and justify.

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{n}{\sqrt{n^3+2}}$

(b) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{n!}{(2n)!}$

12. (10 points) Find the radius of convergence and the interval of convergence of the following power series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{n!}(x+1)^n$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n\sqrt{n}}$

13. (15 points)

(a) Find the Taylor series centered at $a = 0$ of the function $\ln(1 - x^2)$ and its interval of convergence.

(b) Write the integral $\int_0^x \ln(1 - t^2)dt$ as a power series in x, and find its interval of convergence.

14. (10 points)

(a) Compute the Taylor series of the function $f(x) = \sin(x)$ centered at $a = -\pi/2$ and its interval of convergence.

(b) Find the Taylor polynomials of $f(x)$ of degree 2 and 3.

15. (10 points)

(a) Find the first five nonzero terms of the Maclaurin series expansion of the function

$$f(x) = \frac{\cos(x^2) - (1 - \frac{x^4}{2})}{x^8}.$$

(b) What is the value of $f^{(10)}(0)$?

(c) What is the value of $f^{(12)}(0)$?

(d) What is the value of $\lim_{x \to 0} f(x)$?