MATH 162

Final Exam QUESTIONS
December 9, 2004

Part A

1. (30 points)

(a) (10 points) Calculate

$$\int x \sin(x^2)\, dx$$

(b) (10 points) Calculate

$$\int x \ln(x^2)\, dx$$

(c) (10 points) Calculate

$$\int \frac{dy}{y(y^2 - 1)}$$

2. (10 points) Set up a integral which presents the surface area obtained by rotating the curve given by the function below about the line $x = -1$. (First sketch a picture to make sure that you are rotating around the correct line.)

$$y = \frac{1}{3}(x^2 + 2)^{3/2} \qquad 1 \le x \le 2$$

3. (12 points) A hemispherical reservoir is filled with water. The radius of the hemisphere is R ft. The weight of water is 62.5 lb/ft^3. How much work is required to pump the water out of the reservoir until the height(depth) of the water which is left is $R/2$. (Your answer will be in terms of R.)

4. (14 points) A bicycle wheel of radius R rolls along a flat surface at a rate of v feet per second. There is a reflector half way out toward the rim of the wheel. (see figure).

(a) (7 points) Write equations for the x and y coordinates of the reflector as functions of time t.

(b) (7 points) What is the velocity of the reflector in the x direction when it is at its highest point?
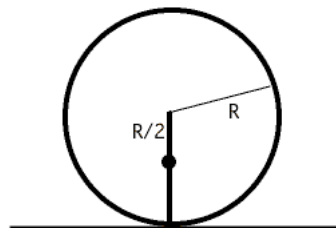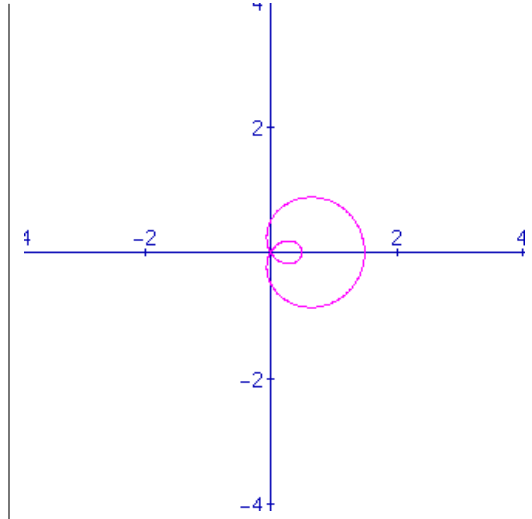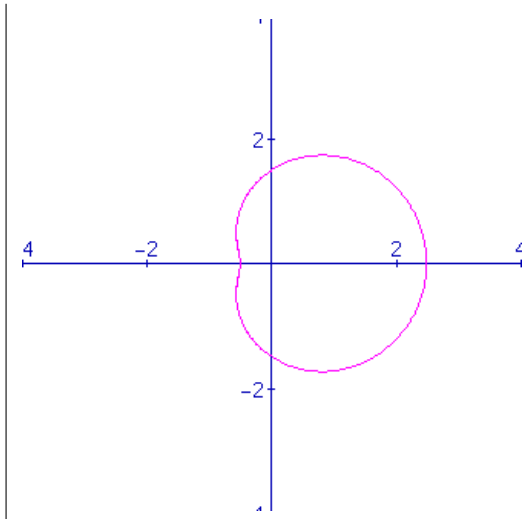

(see figure)

5. (10 points) Make a rough sketch of the graph of $r = \sin(2\theta)$.

Set up an integral which calculates the area of one petal of the graph of $r = \sin(2\theta)$.
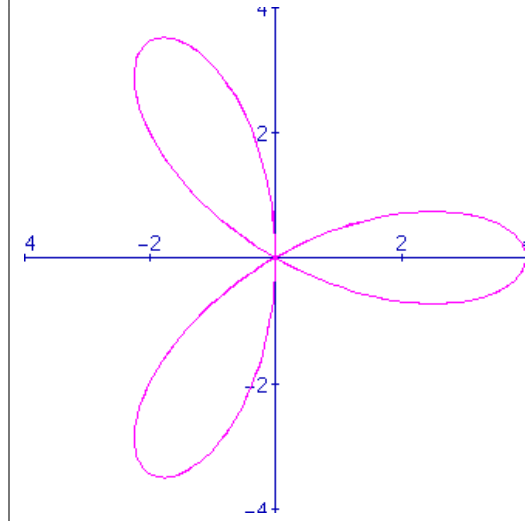
6. (24 points) Match the equations with the graphs. The equations are given on the next page. **Record your answers on the answer sheet, not on this sheet.**
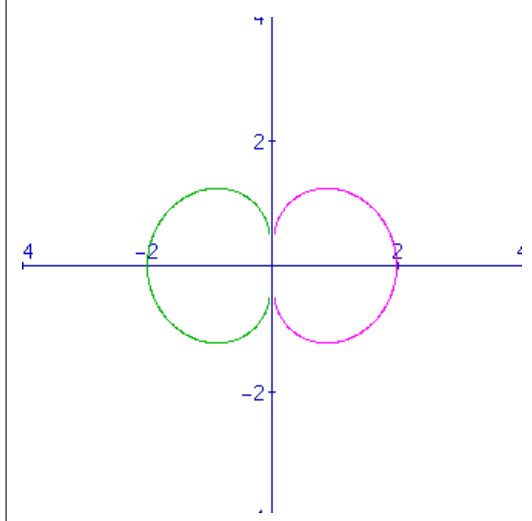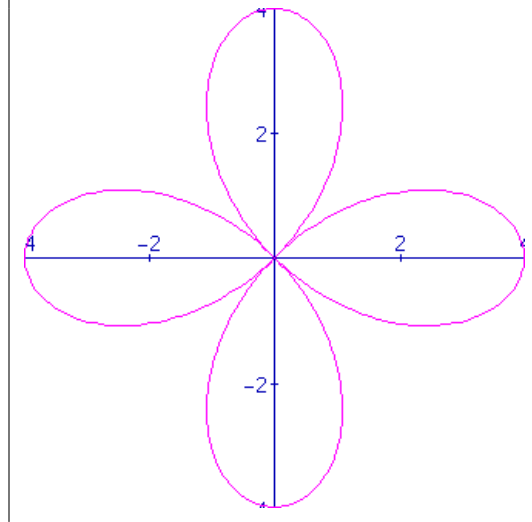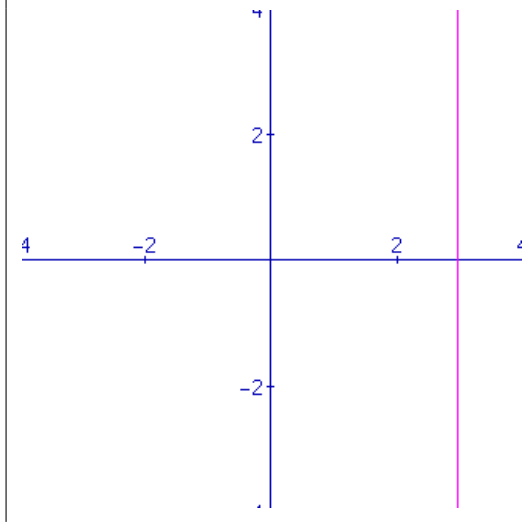
A: _____

B: _____

C: _____

D: _____

E: _____

F: _____

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1. $r^2 = 4\cos^2\theta + \sin^2\theta$
8. $r = 4\cos\theta$

2. $r^2 = 4\sin^2\theta + \cos^2\theta$
9. $r = 4\cos 2\theta$

3. $r = 3/\cos\theta$
10. $r = 4\cos 3\theta$

4. $r = 3/\sin\theta$
11. $r = 4\cos 4\theta$

5. $r = 1.5 + \cos\theta$
12. $r^2 = 4\cos\theta$

6. $r = 1.0 + \cos\theta$
13. $r^2 = 4\cos 2\theta$

7. $r = 0.5 + \cos\theta$
14. $r^2 = 4\cos 3\theta$

Part B

1. (33 points) (a) (11 points) Is the following series convergent or divergent? You must justify your answer.

$$\sum_{n=1}^{\infty} \frac{n^2 + 3}{2n^2 + 4n + 7}$$

(b) (11 points) Is the following series convergent or divergent? You must justify your answer.

$$\sum_{n=1}^{\infty} \frac{n^2 + 4}{n^3 + 5n}$$

(c) (11 points) Is the following series convergent or divergent? You must justify your answer.

$$\sum_{n=1}^{\infty} \frac{1}{2^{(n^2)}}$$

2. (11 points) Is the following series absolutely convergent, conditionally convergent, or divergent? You must justify your answer.

$$1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \ldots$$

3. (11 points) Find the radius of convergence for the following power series.

$$\sum_{n=1}^{\infty} \frac{n^2}{2^n} x^n$$

4. (11 points) Find the Taylor polynomial

$$T_2(x) = c_0 + c_1 x + c_2 x^2$$

for

$$f(x) = \sqrt{1 + e^x}.$$

5. (12 points) Suppose that

$$f^{(5)}(x) = \frac{\cos^2 x}{e^x + 3}.$$

(a) (6 points) Find a number M such that

$$|f^{(5)}(x)| \leq M$$

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for all x.

(b) (6 points) For which x can you guarantee that the error $|R_4(x)|$ is less than 10^{-7}?

Find a bound for the remainder term $|R_4(x)|$, valid for all values of x.

6. (11 points) Find the Taylor series $c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots$ for

$$f(x) = xe^{x^3}.$$

(Write out at least the first three non-zero terms.)

7. (11 points) The Taylor series for $f(x) = x \sin(x^2)$ is

$$f(x) = x^3 - \frac{x^7}{3!} + \frac{x^{11}}{5!} - \dots$$

Find $f^{(11)}(0)$.