Math 162: Calculus IIA

Final Exam

December 16, 2010

NAME (please print legibly): _____

Your University ID Number: _____

Indicate your instructor with a check in the box:

Ang Wei	MWF 9:00 - 9:50 AM	
Doug Ravenel	MWF 10:00 - 10:50 AM	
Jon Carstea	MW 2:00 - 3:15 PM	

- **The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **Put your answers in the space provided at the bottom of each page or half page.**
- **You are responsible for checking that this exam has all 17 pages.**
- **Part A covers the same material as the two midterms, and Part B covers additional material. Letter grades will be computed for the two parts separately.**
- **Part B will count for 20% of your course grade. It has the same weight as a midterm exam grade.**
- **Part A will count for at least 10% of your course grade.**
- **If your grade on part A is better than your lowest midterm exam grade, then it will replace that midterm exam grade and count for 30% of your course grade.**
- ***Have a nice winter break!***

Part A		
QUESTION	VALUE	SCORE
1	20	
2	20	
3	15	
4	15	
5	20	
6	10	
TOTAL	100	

Part B		
QUESTION	VALUE	SCORE
7	20	
8	20	
9	20	
10	20	
11	20	
TOTAL	100	

Part A

1. (20 points)

(a) Find the partial fraction expansion of

$$\frac{1}{x^3 - x}.$$

ANSWER:

(b) Calculate the integral

$$\int \frac{1}{x^3 - x}\, dx.$$

NOTE: The first part of this problem was designed to help you do the second part. If you did the first part incorrectly, you will not get partial credit for "correctly" using the wrong partial fraction expansion to find the integral.

ANSWER:

2. (20 points)

Evaluate the integral

$$\int \frac{x+1}{\sqrt{x^2+4}}\,dx$$

ANSWER:

3. (15 points)

Rotate the region bounded by $y = 0$, $y = \sin x$, $x = 0$ and $x = \pi$ around the x-axis. Compute the volume of the resulting body.

ANSWER:

4. (15 points)

An inverted cone has radius of the base $2m$ and depth/height $5m$. The cone if filled with water up to the height of $3m$. How much work (in Joules) is required to empty the cone? The density of water is $1000kg/m^3$ and $g = 9.8m/sec^2$. You may assume $9.8 \cdot \pi = 31$. A Joule is the metric unit of work, $1J = 1kg \cdot m^2/sec^2$

ANSWER:

5. (20 points)

The cardioid is the curve defined in polar coodinates by $r = 1 + \cos\theta$. Find the arclength of the cardioid for $0 \leq \theta \leq \pi$. You may use the identity $\cos^2(\theta/2) = (1 + \cos\theta)/2$.

ANSWER:

6. (10 points)

Find the area of the region bounded by $y = \sin(x)$ and $y = \cos(x)$ for $\pi/4 \leq x \leq 5\pi/4$.

ANSWER:

Part B

7. (20 points)

(a) Find the Maclaurin series expansion of $\sin x/x$, as well as the interval of convergence.

ANSWER:

b) Find the Maclaurin series for

$$\int_0^x \frac{\sin t}{t} dt,$$

as well as the interval of convergence.

ANSWER:

8. (20 points)

(a) Find the Taylor series centered at 0 of the function $\sin(x^2)$, as well as radius and interval of convergence.

ANSWER:

(b) Write the integral

$$\int_0^x \sin(t^2)dt$$

as a power series in x.

ANSWER:

9. (20 points) Find the radius of convergence and interval of convergence of the series

$$\sum_{n=2}^{\infty}(-1)^n\frac{x^n}{4^n\ln(n)}\,.$$

ANSWER:

10. (20 points)

Determine whether the series

$$\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n^2+1}}{n^2}$$

is absolutely convergent, conditionally convergent or divergent.

ANSWER:

11. (20 points)

(a) Determine whether the series

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{n^n}$$

is absolutely convergent, conditionally convergent or divergent.

ANSWER:

(b) Estimate the sum of the series within an accuracy of $\frac{1}{16}$.

ANSWER: