Math 162: Calculus IIA

Final Exam

December 15, 2019

NAME (please print legibly): ____________________

Your University ID Number: ____________________

Your University email ____________________

Indicate your instructor with a check in the box:

Saul Lubkin	MW 9:00 - 10:15 AM	
Doug Ravenel	MWF 10:25 - 11:40 AM	
Charles Wolf	MW 12:30 - 1:45 PM	
Rufei Ren	MW 4:50 - 6:05 PM	

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam and that all work will be my own.

Signature: ____________________

- **The presence of calculators, cell phones and other electronic devices at this exam is strictly forbidden and WILL BE TREATED AS AN ACADEMIC HONESTY VIOLATION.**
- **Show your work and justify your answers. Put your answers in the space provided at the bottom of each page or half page. SIMPLIFY YOUR ANSWERS AS MUCH AS POSSIBLE.**
- **Part A (problems 1–7) covers the same material as the two midterms, and Part B (problems 8–11) covers additional material. Letter grades will be computed for the two parts separately. Part B will count for 20% of your course grade. Part A will count for at least 10% of your course grade. If your letter grade on part A is better than your lowest midterm letter exam grade, then it will replace that midterm exam grade and count for 30% of your course grade.**
- **You are responsible for checking that this exam has all 24 pages.**

Part A

1. (20 points) Compute the following integral:

$$\int \frac{x+3}{(x-1)^2(x+2)} dx$$

ANSWER:

2. (20 points)

Consider the function $y = \sqrt{x+1}$ on the interval $[1, 5]$.

(a) **(10 Points)** Compute the volume of the region bound by the curves $y = \sqrt{x+1}$, $x = 1, x = 5$ and the x-axis, revolved about the x-axis.

ANSWER:

(b) **(10 Points)** Compute the surface area of the region bound by the curves $y = \sqrt{x+1}$, $x = 1$, and $x = 5$, revolved about the x-axis.

ANSWER:

3. (20 points)

Find the arc length L of the parametric curve, $x = e^t \cos t$, $y = e^t \sin t$, from $t = 0$ to $t = \pi$.

ANSWER:

4. (20 points)

(a) Compute the volume of a region bounded by the curves $y = x^4 + 1$, $y = 1$ and $x = 1$ and rotated around the y-axis.

ANSWER:

(b) Set up the integral for the volume of the region bounded by $y = x^3$, $y = 0$ and $x = 2$ and rotated around line $x = 2$. Use the shell method. Do not evaluate the integral.

ANSWER:

5. (10 points)

Evaluate the integral

$$\int \ln(x^{\frac{1}{2}})dx.$$

ANSWER:

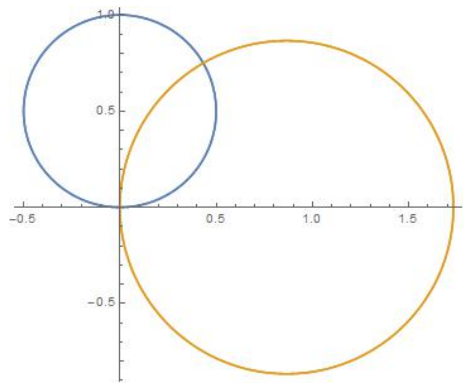
6. (20 points) Compute

$$\int \frac{x^2}{(1-x^2)^{3/2}} dx$$

ANSWER:

7. (20 points)

(a) Find the area of the region both inside the circle $r = \sin\theta$ and outside the circle $r = \sqrt{3}\cos\theta$ (both equations are in polar coordinates). The two circles are shown below. THEY INTERSECT AT THE ORIGIN AND THE POLAR POINT $(\theta, r) = (\pi/3, \sqrt{3}/2)$.



ANSWER:

(b) Compute the equation (in Cartesian coordinates x, y) of the tangent line to the circle $r = \sin\theta$ at the points where it intersects the circle $r = \sqrt{3}\cos\theta$

ANSWER:

Part B

8. (20 points)

Find the radius of convergence and interval of convergence of the series

$$\sum_{n=2}^{\infty} \frac{\pi^n (x-2)^n}{\ln n}.$$

ANSWER:

9. (20 points)

(a) Is the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

Absolutely convergent, conditionally convergent or divergent? Justify your answer.

ANSWER:

(b) The series

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2}$$

converges absolutely. How many terms do you have to add to estimate the sum with an accuracy of 1/100?

ANSWER:

10. (20 points)

(a) Find a power series expansion for the function $f(x) = x^2 e^{-x^2}$ centered at $x = 0$.

ANSWER:

(b) Find the radius of convergence for the series you found in part (a).

ANSWER:

(c) Compute $f^{(6)}(0)$ and $f^{(2019)}(0)$.

ANSWER:

11. (20 points)

(a) **(10 Points)** Show that the following series converges:

$$\sum_{n=1}^{\infty} \frac{1}{n \cdot 3^n}$$

ANSWER:

(b) **(5 Points)** Find the Maclaurin power series representation for $-\ln|1-x|$. (Hint: What is the Maclaurin series for $1/(1-x)$?)

ANSWER:

(c) **(5 Points)** What is the value of this series:

$$\sum_{n=1}^{\infty} \frac{1}{n \cdot 3^n}?$$

ANSWER:

This is scratch paper. If you use it to work on a problem, please indicate so on the page where that problem occurs.

Second scratch paper page. If you use it to work on a problem, please indicate so on the page where that problem occurs.

More scratch paper. If you use it to work on a problem, please indicate so on the page where that problem occurs.

And even more scratch paper. If you use it to work on a problem, please indicate so on the page where that problem occurs.