Math 162: Calculus IIA

Final Exam

December 17, 2016

NAME (please print legibly): _____

Your University ID Number: _____

Your University email _____

Indicate your instructor with a check in the box:

Jie Zhong	MWF 9:00 - 9:50 AM	
Doug Ravenel	MWF 10:25 - 11:15 AM	
Doug Haessig	MW 12:30 - 1:45 PM	
Carl McTague	MW 4:50-6:05 PM	

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam and that all work will be my own.

Signature: _____

- **The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden and WILL BE TREATED AS AN ACADEMIC HONESTY VIOLATION.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given. Put your answers in the space provided at the bottom of each page or half page. SIMPLIFY YOUR ANSWERS AS MUCH AS POSSIBLE.**
- **You are responsible for checking that this exam has all 20 pages.**
- **Part A covers the same material as the two midterms, and Part B covers additional material. Letter grades will be computed for the two parts separately. Part B will count for 20% of your course grade. It has the same weight as a midterm exam grade. Part A will count for at least 10% of your course grade. If your grade on part A is better than your lowest midterm exam grade, then it will replace that midterm exam grade and count for 30% of your course grade.**

Part A		
QUESTION	VALUE	SCORE
1	15	
2	20	
3	10	
4	20	
5	15	
6	20	
TOTAL	100	

Part B		
QUESTION	VALUE	SCORE
7	20	
8	20	
9	20	
10	20	
11	20	
TOTAL	100	

Part A

1. (15 points) Evaluate the integral

$$\int \frac{x^3}{\sqrt{4-x^2}}\,dx$$

ANSWER:

2. (20 points)

(a) Compute the volume of a region bounded by the curves $y = x^3 + 1$, $y = 1$ and $x = 1$ and rotated around the y-axis.

ANSWER:

(b) Set up the integral for the volume of the region bounded by $y = x^4$, $y = 0$ and $x = 2$ and rotated around line $x = 2$. Use the shell method. Do not evaluate the integral.

ANSWER:

3. (10 points)

Evaluate the integral

$$\int \arcsin x \, dx.$$

ANSWER:

4. (20 points)

(a) Find the partial fraction decomposition of

$$\frac{x^2+3x}{x^2-1}.$$

ANSWER:

(b) Write out the form of the partial fraction decomposition of the function

$$\frac{x^3 - 2}{(x+1)^3(x^2+1)^2(x-1)} = \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

Do not determine the numerical values of the coefficients.

ANSWER:

(c) Let

$$f(x) = \frac{1}{x-1} + \frac{2x+3}{x^2+1}.$$

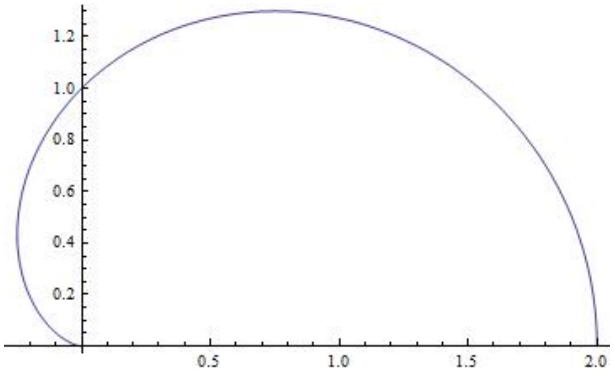
Evaluate

$$\int f(x)dx.$$

ANSWER:

5. (15 points)

The cardioid is the curve defined in polar coordinates by $r = 1 + \cos\theta$. Find the area of the region bounded above by the cardioid and below by the x-axis.



ANSWER:

6. (20 points)

Find the arc length of the parametric curve $x(t) = e^t \cos t$, $y(t) = e^t \sin t$ connecting the point $(1, 0)$ to the point $(e^{2\pi}, 0)$.

ANSWER:

Part B

7. (20 points)

(a) Find a power series representation centered at 1 as well as the radius and interval of convergence for the function

$$f(x) = \frac{x-1}{x+2}.$$

ANSWER:

(b) Write the following integral as a power series in $x-1$. What is the radius of convergence of this power series?

$$\int \frac{x-1}{x+2} dx$$

ANSWER:

8. (20 points)

Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n}\ln n}$$

ANSWER:

9. (20 points)

Find the radius of convergence and interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{3^n(x-2)^n}{\sqrt[3]{n}}.$$

ANSWER:

10. (20 points) Let

$$f(x) = \frac{x^2}{1+2x}.$$

(a) Find the Taylor series of $f(x)$ centered at $x = 0$.

ANSWER:

(b) Find the radius of convergence.

ANSWER:

(c) Compute $f^{(100)}(0)$.

ANSWER:

11. (20 points)

(a) Determine whether the series

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n^5}$$

is absolutely convergent, conditionally convergent, or divergent.

ANSWER:

(b) Estimate the sum of the series with an accuracy of $.01 = 1/100$.

ANSWER: