Math 162: Calculus IIA

Final Exam

December 15, 2015

NAME (please print legibly): _____

Your University ID Number: _____

Your University email _____

Indicate your instructor with a check in the box:

JJ Lee	MWF 9:00 - 9:50 AM	
Doug Ravenel	MWF 10:25 - 11:15 AM	
Timur Akhunov	MW 12:30 - 1:45 PM	
Eyal Neuman	MW 4:50-6:05 PM	

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam and that all work will be my own.

Signature: _____

- **The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given. Put your answers in the space provided at the bottom of each page or half page.**
- **You are responsible for checking that this exam has all 20 pages.**
- **Part A covers the same material as the two midterms, and Part B covers additional material. Letter grades will be computed for the two parts separately. Part B will count for 20% of your course grade. It has the same weight as a midterm exam grade. Part A will count for at least 10% of your course grade. If your grade on part A is better than your lowest midterm exam grade, then it will replace that midterm exam grade and count for 30% of your course grade.**
- ***Have a nice winter break!***

Part A		
QUESTION	VALUE	SCORE
1	15	
2	20	
3	10	
4	20	
5	15	
6	20	
TOTAL	100	

Part B		
QUESTION	VALUE	SCORE
7	20	
8	20	
9	20	
10	20	
11	20	
TOTAL	100	

Part A

1. (15 points) Evaluate the integral

$$\int \frac{1}{x^2\sqrt{x^2+16}}\,dx.$$

ANSWER:

2. (20 points)

(a) Compute the volume of a region bounded by the curves $y = x^3 + 1$, $y = 1$ and $x = 1$ and rotated around the x-axis.

ANSWER:

(b) Set up the integral for the volume of the region bounded by $y = x^4$, $y = 0$ and $x = 2$ and rotated around the x-axis. Use the washer method. Do not evaluate the integral.

ANSWER:

3. (10 points)

Evaluate the integral

$$\int (\ln x)^2 \, dx.$$

ANSWER:

4. (20 points)

(a) Find the partial fraction decomposition of

$$\frac{3x-2}{x^2-x}.$$

ANSWER:

(b) Write out the form of the partial fraction decomposition of the function

$$\frac{2+x^3}{x^5+2x^3+x} = \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

Do not determine the numerical values of the coefficients.

ANSWER:

(c) Let

$$f(x) = \frac{1}{x} + \frac{4x+5}{x^2+1}.$$

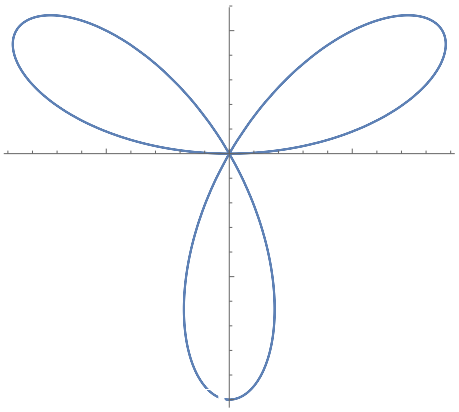
Evaluate

$$\int f(x)dx.$$

ANSWER:

5. (15 points)

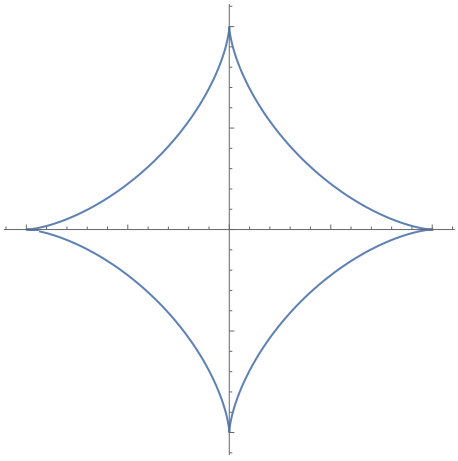
Use the polar area formula to find the area of one leaf of the three leafed rose, the polar curve defined by $r = \sin 3\theta$, that is the area for $0 \leq \theta \leq \pi/3$.



ANSWER:

6. (20 points)

Find the arc length of the astroid, the parametric curve defined by $x = \cos^3 t$ and $y = \sin^3 t$ for $0 \leq t \leq 2\pi$.



ANSWER:

Part B

7. (20 points)

(a) Find a power series representation centered at 1 as well as the radius and interval of convergence for the function

$$f(x) = \frac{2(x-1)}{1+2(x-1)^2}.$$

ANSWER:

(b) Write the following integral as a power series in x. What is the radius of convergence of this power series?

$$\int \frac{2(x-1)}{1+2(x-1)^2}dx$$

ANSWER:

8. (20 points)

Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n - 2\sqrt{n} + 2}$$

ANSWER:

9. (20 points)

Find the radius of convergence and interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{4^n(n+1)}.$$

ANSWER:

10. (20 points)

(a) Find the Taylor series centered at 0 of the function

$$g(x) = \tan^{-1}(x^2) - x^2,$$

as well as the radius of convergence.

ANSWER:

(b) Write the derivative of $g(x)$ as a power series and use it to calculate

$$\left.\frac{dg(x)}{dx}\right|_{x=0}$$

ANSWER:

11. (20 points)

(a) Determine whether the series

$$\sum_{n=1}^{\infty}(-1)^n\frac{n!}{n^n}$$

is absolutely convergent, conditionally convergent, or divergent.

Hint: You may use the fact that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$.

ANSWER:

(b) Estimate the sum of the series with an accuracy of $\frac{1}{10}$.

ANSWER: