Math 162: Calculus IIA

Final Exam
December 17, 2011

NAME (please print legibly): _____

Your University ID Number: _____

Indicate your instructor with a check in the box:

Don Larson	MWF 9:00 - 9:50 AM	
Doug Ravenel	MWF 10:00 - 10:50 AM	
Yoonbok Lee	MWF 11:00 - 11:50 AM	

- The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.

- Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.

- Put your answers in the space provided at the bottom of each page or half page.

- You are responsible for checking that this exam has all **17** pages.

- Part A covers the same material as the two midterms, and Part B covers additional material. Letter grades will be computed for the two parts separately.

- Part B will count for **20%** of your course grade. It has the same weight as a midterm exam grade.

- Part A will count for at least **10%** of your course grade.

- If your grade on part A is better than your lowest midterm exam grade, then it will replace that midterm exam grade and count for **30%** of your course grade.

- *Have a nice winter break!*

Part A		
QUESTION	VALUE	SCORE
1	20	
2	20	
3	15	
4	15	
5	20	
6	10	
TOTAL	100	

Part B		
QUESTION	VALUE	SCORE
7	20	
8	20	
9	20	
10	20	
11	20	
TOTAL	100	

Part A

1. (20 points)

(a)   Find the partial fraction expansion of

$$\frac{1}{x^3 - x^2 - 6x}.$$

ANSWER:

(b)   Calculate the integral

$$\int \frac{1}{x^3 - x^2 - 6x}\, dx.$$

NOTE: The first part of this problem was designed to help you do the second part. If you did the first part incorrectly, you will not get partial credit for "correctly" using the wrong partial fraction expansion to find the integral.

ANSWER:

2. (20 points) Evaluate the integral

$$\int \frac{x^2}{\sqrt{16 - x^2}} \, dx$$

[Hint: You may find the identity $\sin(2\theta) = 2\sin\theta\cos\theta$ useful.]

ANSWER:

3. (15 points) Find the volume of the solid obtained by rotating the region bounded by the curves $y = x^4$, $y = 0$, and $x = 1$ about the line $x = 2$.
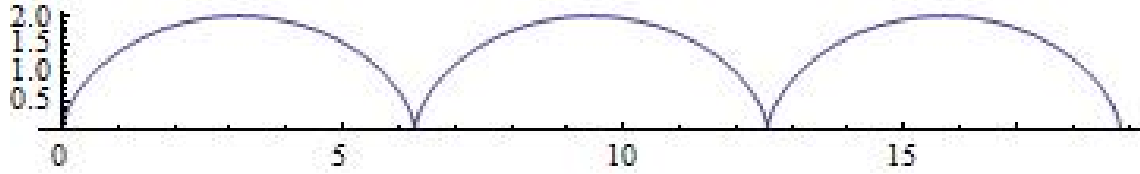
ANSWER:

4. (15 points)

Evaluate the integral
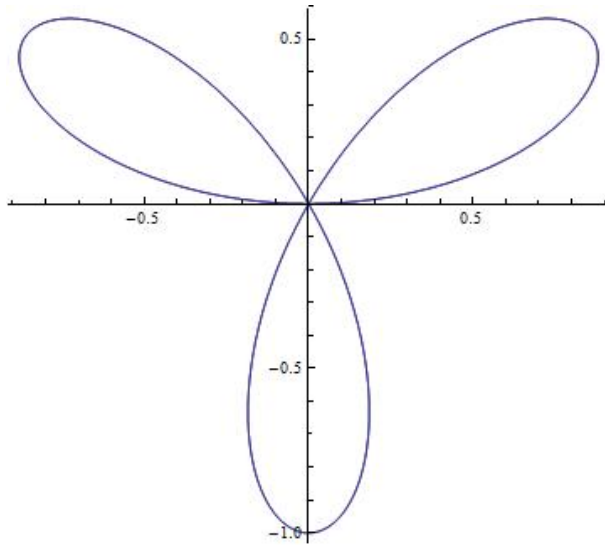
$$\int \cos x \ln(\sin x)\, dx$$

ANSWER:

5. (20 points) Find the arc-length of the cycloid $x = t - \sin(t)$ and $y = 1 - \cos(t)$, $t \in [0, 6\pi]$. HINT: You may use the identity $\sqrt{(1 - \cos t)/2} = |\sin(t/2)|$. It is easier to find the length of one arch and triple it.

ANSWER:

6. (10 points)

Find the area enclosed by one leaf in the three leafed rose defined in polar coordinates by $r = \sin 3\theta$.

ANSWER:

Part B

7. (20 points)

(a) Find the Maclaurin series expansion of $(\arctan x)/x$, as well as the interval of convergence.

ANSWER:

b)   Find the Maclaurin series for

$$\int_0^x \frac{\arctan t}{t}\, dt,$$

as well as the interval of convergence.

ANSWER:

8. (20 points)

(a)   Find the Taylor series centered at 0 of the function $\ln(1 - x^3)$, as well as radius and interval of convergence.

ANSWER:

(b)   Write the integral

$$\int_0^x \ln(1 - t^3)dt$$

as a power series in x.

ANSWER:

9. (20 points) Find the radius of convergence and interval of convergence of the series

$$\sum_{n=2}^{\infty}(-1)^n\frac{x^n}{2^n n(\ln n)^2} \ .$$

ANSWER:

10. (20 points)

Determine whether the series

$$\sum_{n=2}^{\infty}(-1)^n\frac{1}{n(\ln n)^3}$$

is absolutely convergent, conditionally convergent or divergent.

ANSWER:

11. (20 points)

(a) Determine whether the series
$$\sum_{n=1}^{\infty} \frac{(-2)^n}{(n!)^2}$$
is absolutely convergent, conditionally convergent or divergent.

ANSWER:

(b) Estimate the sum of the series within an accuracy of $\frac{1}{36}$.

ANSWER: