Part I

1. Compute the following integrals.

(a) $\displaystyle\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx$

(b) $\displaystyle\int \frac{3x+1}{x^2+x-2}\,dx$

(c) $\displaystyle\int xe^x\,dx$

(d) $\displaystyle\int \sin^2(3x+2)\,dx$

(e) $\displaystyle\int \sin^3(x)\cos^2(x)\,dx$

2.

(a) Find the area of the region enclosed by $y = x^3 - x$ and $y = 3x$.

(b) Find the volume of the solid obtained by rotating the region bounded by $y = 3 + 2x - x^2$ and $y = 3 - x$ about the line y-axis.

(c) Find the volume of the solid obtained by rotating the region bounded by $y = x$ and $y = \sqrt{x}$ about the line $y = 1$.

(d) A spring has a natural length of 20cm. If a 25-N force is required to keep it stretched to a length of 30 cm, how much work is required to stretch it from 20cm to 25cm?.

3. Three improper integrals are given below. Indicate whether they are convergent or divergent and evaluate those which are convergent.

(a) $\displaystyle\int_{-1}^{1} \frac{e^x}{e^x - 1}\, dx$

(b) $\displaystyle\int_{1}^{\infty} \frac{\ln(x)}{x^2}\, dx$

(b) $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{9 + x^6}\, dx$

4.

(a) Find the length of the curve $y = \ln(x)$, $1 \le x \le \sqrt{3}$.

(b) Rotate the curve $y = \sqrt{x}$, $4 \le x \le 9$ about the x-axis. Find the surface area.

(c) Consider the curve given by $x(t) = e^t$, $y(t) = (t-1)^2$. Find the tangent line at time $t = 0$.

(d) Set up an integral giving the length of the curve in (c) from $0 \le t \le 4$.

(e) Set up an integral giving the surface area when the curve in (c) from $0 \le t \le 4$ is rotated about the y-axis.

5. Consider the curve given in polar coordinates by the equation $r = 1 + \cos(\theta)$.

(a) Give an accurate sketch of this curve.

(b) Find the area enclosed by one loop of $r = 3\cos(5\theta)$.

Part II

6. Consider the following geometric series. Find their sum if they converge or write "divergent" otherwise.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-6)^{n-1\,n}}{5^n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}}$

7. Determine whether each of the following series is Absolutely Convergent (AC), converges but is not absolutely convergent, i.e. is Conditionally Convergent (CC), or is Divergent (D) and give a short reason why. For example, $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$ is D by comparison with the Harmonic series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^k k!}{(k+2)!}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-1}{n^2+1}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln(n)^2}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3+3n^2}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^{2n}}{n^n}$

8. Consider the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n^2 5^n}$

(a) Find the radius of convergence of this power series.

(b) Find the interval of convergence of this power series (be sure to check endpoints).

9. Each of the functions below has a Taylor series about $x = 0$. Find the Taylor series.

(a) $\dfrac{\cos(x) - 1}{x^2}$

(b) $\dfrac{x}{1 + x^3}$

(c) $\displaystyle\int \sin(x^2)\, dx$

(d) $\dfrac{d}{dx} x e^{x^3}$

(e) $\ln(1 - x)$

(f) $\arctan(x)$

10.

(a) Write down the general form of the Taylor series of a function $f(x)$ at $x = a$ (or about $x = a$ or centered at $x = a$).

(b) Write down the Taylor series for $f(x) = \ln(x)$ at $x = 5$. You can either use summation notation or write down the first 5 non-zero terms.