Math 162: Calculus IIA

Final Exam QUESTIONS

April 30, 2007

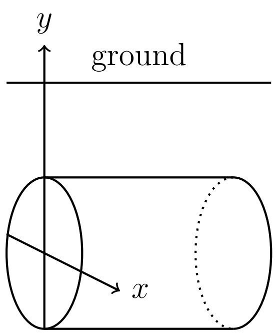
Part I

1. (10 points)

(a) Find the area enclosed by the curves $y = x + 2$ and $y = x^2$.

(b) Find the volume of the solid obtained by rotating this same region about the x-axis.

2. (10 points) Gasoline at a service station is stored in a cylindrical tank buried on its side, with the highest part of the tank 5 ft below the surface. The tank is 8 feet in diameter and 10 ft long. The density of gasoline is 45 lb/ft^3. Assume that the filler cap of each automobile is 2 feet above the ground. If the tank is initially full, how much work is done pumping half of the gasoline in the tank into automobiles? (You *do not need to multiply out your answer*, but it should be simplified otherwise.)



3. (10 points) Find the definite integral

$$\int_0^{2\pi} x \sin x \, dx$$

4. (10 points) Solve this integral:

$$\int \frac{x^2}{(\sqrt{16 - x^2})^3} \, dx$$

5. (10 points) Evaluate this integral:

$$\int_{-3}^{-1} \frac{1}{x(2x+1)} \, dx$$

6. (10 points)

Does the following series converge or diverge? Why or why not?

$$\sum_{n=1}^{\infty} \frac{(-1)^n \ln(n)}{n}$$

7. (10 points)

(a) Find the limit $\lim_{n\to\infty} \dfrac{2n^2+n-3}{5n^2+2}$.

(b) Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2+n-3}{5n^2+2}$ converge or diverge?

8. (10 points) Evaluate $\displaystyle\int_1^{\infty} \frac{x^3}{(x^4+1)^{10}}\,dx$.

9. (10 points) Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{(n^4+1)^{10}}$ converge or diverge?

Part II

10. (10 points) The power series for $f(x) = \dfrac{1}{1+x^2}$ is given by $\dfrac{1}{1+x^2} = \displaystyle\sum_{n=1}^{\infty}(-1)^n x^{2n}$. Use this to get the power series of $\arctan(x)$.

11. (10 points) Consider the series

$$\begin{aligned}\sum_{n=0}^{\infty} \frac{(-2x)^{3n}}{n!} &= 1 - 2^3x^3 + \frac{2^6x^6}{2!} - \frac{2^9x^9}{3!} + \frac{2^{12}x^{12}}{4!} + \cdots \\ &= 1 - 8x^3 + 32x^6 - \frac{256x^9}{3} + \frac{512x^{12}}{3} + \cdots\end{aligned}$$

For which values of x does it converge?

12. (10 points) Suppose that for the power series $\displaystyle\sum_{n=0}^{\infty} c_n x^n$ centered at $a = 0$, we know $\displaystyle\sum_{n=0}^{\infty} c_n 2^n$ converges and $\displaystyle\sum_{n=0}^{\infty} c_n(-4)^n$ diverges. Then for each of the following series state if it converges, diverges or it is unknown. Justify your answers.

(a) $\displaystyle\sum_{n=0}^{\infty} c_n$

(b) $\sum_{n=0}^{\infty} c_n(-2)^n$

(c) $\sum_{n=0}^{\infty} c_n 5^n$

13. (10 points)

(a) Find $T_3(x)$, the third degree Taylor polynomial for $f(x) = \frac{1}{x}$ at $a = 1$.

(b) Use Taylor's inequality to estimate the error when $T_3(x)$ is used as an approximation for $f(x)$ on the interval $\frac{1}{2} \leq x \leq \frac{3}{2}$.

14. (10 points) Consider the curve defined by the parametric equations

$$x = t^2 \qquad \text{and} \qquad y = 3t - t^3.$$

(a) For which values of t is the tangent line vertical? Find the corresponding points.

(b) For which values of t is the tangent line horizontal? Find the corresponding points.

15. (10 points)

Set up (but do not evaluate) the integral to find the length of the curve $x = \frac{1}{3}\sqrt{y}(y-3)$ for $4 \leq y \leq 9$.

16. (10 points) Find the area of the surface obtained rotating the curve $y = \sqrt{x}$, $0.75 \leq x \leq 3.75$, about the x-axis.

17. (10 points) Consider the polar curve

$$r = 1 + \theta^2, \qquad 0 \leq \theta \leq 2\pi$$

Find the area of the region bounded by the curve and the ray $\theta = 0$, $r \geq 0$.

18. (10 points) Find the arclength of the curve defined in polar coordinates by the equation

$$r = a \sin\theta$$

where a is a positive constant and $0 \leq \theta \leq \pi$.