Math 162: Calculus IIA

Final Exam

December 16, 2007

NAME (please print legibly): _____

Your University ID Number: _____

Indicate your instructor with a check in the box:

Juan Ortiz-Navarro	MWF 9:00 - 9:50 AM	
Doug Ravenel	MWF 10:00 - 10:50 AM	

- **The presence of calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.**
- **Put your answers in the space provided at the bottom of each page.**
- **You are responsible for checking that this exam has all 15 pages.**
- **Part A covers the same material as the two midterms, and Part B covers additional material. Letter grades will be computed for the two parts separately.**
- **Part B will count for 20% of your course grade. It has the same weight as a midterm exam grade.**
- **Part A will count for at least 10% of your course grade.**
- **If your grade on part A is better than your lowest midterm exam grade, then it will replace that midterm exam grade and count for 30% of your course grade.**

Part A		
QUESTION	VALUE	SCORE
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
TOTAL	60	

Part B		
QUESTION	VALUE	SCORE
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
TOTAL	70	

Part A

1. (10 points)

A circular swimming pool has a diamater of 24 ft., the sides are 5 ft. high, and the depth of the water is 4 ft. How much work is required to pump all of the water out over the side? (Use the fact that water weighs 62.5 lb/ft^3.)

ANSWER: _____

2. (10 points)

Find the definite integral

$$\int_0^{\frac{\pi}{2}} x\cos(2x)\,dx$$

ANSWER: _____

3. (10 points)

Solve this integral:

$$\int \frac{\sqrt{9-x^2}}{x^2}\,dx$$

ANSWER: _____

4. (10 points)

Evaluate this integral:

$$\int \frac{1}{x^2 + x}\, dx$$

ANSWER: _____

5. (10 points)

(a) Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$ converge or diverge? Why?

ANSWER: _____

(b) Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{5n^2+4}$ converge or diverge? Why?

ANSWER: _____

6. (10 points) Find all x for which the following power series converges, i.e. find the interval of convergence:

$$\sum_{n=1}^{\infty} \frac{-1^n}{n+1}(x+1)^n$$

ANSWER: _____

Part B

7. (10 points)

The power series for e^{-x^2} is given by

$$\begin{aligned} e^{-x^2} &= \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{n!} \\ &= 1 - x^2 + \frac{x^4}{2} - \frac{x^6}{6} + \frac{x^8}{24} + \cdots \end{aligned}$$

Use this to get the power series for the area under the bell curve,

$$f(x) = \int_0^x e^{-t^2}dt.$$

You can either use summation notation or write down the first 5 non-zero terms.

ANSWER: _____

8. (10 points)

(a) Find $T_3(x)$, the third degree Taylor polynomial for $f(x) = \sqrt{x}$ at $a = 4$.

ANSWER: _____

(b) Use Taylor's inequality to find the largest integer k such that the error when $T_3(x)$ is used as an approximation for $f(x)$ on the interval $4 \leq x \leq 5$ is less than 10^{-k}.

ANSWER: _____

9. (10 points) (a) Write the general formula for the Taylor series of a function $f(x)$ at a (or "about a" or "centered at a").

ANSWER: _____

(b) Write the Taylor series of $f(x) = e^{2x}$ at $a = 1$. You can either use summation notation or write down the first 5 non-zero terms.

ANSWER: _____

10. (10 points) Consider the cycloid defined by the parametric equations

$$x = 2(t - \sin t) \qquad \text{and} \qquad y = 2(1 - \cos t).$$

for $0 \leq t \leq 2\pi$.

(a) For which values of t is the tangent line vertical? Find the corresponding points.

ANSWER: _____

(b) For which values of t is the tangent line horizontal? Find the corresponding points.

ANSWER: _____

11. (10 points) Find the length of the cycloid of the previous problem for $0 \leq t \leq \pi$.

Hint: Use the half angle formula $\sin(\theta/2) = \sqrt{(1-\cos\theta)/2}$.

ANSWER: _____

12. (10 points)

Find the area of the surface obtained rotating the semicircle $y = \sqrt{25 - x^2}$, $3 \leq x \leq 4$, about the x-axis.

ANSWER: _____

13. (10 points)

Find the area enclosed by the 8-leafed rose defined by $r = \sin 4\theta$ for $0 \leq \theta \leq 2\pi$.

ANSWER: _____