MATH 162

FINAL EXAM QUESTIONS

April 25, 2007

Part A

Important formulas:

$$(\sin x)' = \cos x \qquad (\cos x)' = -\sin x$$
$$(\tan x)' = \sec^2 x \qquad (\cot x)' = -\csc^2 x$$
$$(\sec x)' = \tan x \sec x \qquad (\csc x)' = -\cot x \csc x$$

$$\begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ \tan^2 x + 1 &= \sec^2 x \end{aligned}$$

1. (11 points) Find the volume of the solid obtained by rotating about the y-axis the region under the curve $y = \sqrt{1 - x^2}$, for $1/2 \leq x \leq 1$.

2. (11 points) A large bathtub has the shape of a hemisphere (half of a sphere) of radius 5 feet, with the center at ground level. It is full of water from the bottom to ground level. How much work is done in pumping the water to the top? Remember that the weight of water is 62.4 pounds per cubic foot.

3. (11 points) Solve this indefinite integral:

$$\int x^{3/2} \ln x \, dx$$

4. (11 points) Solve this integral:

$$\int \frac{1}{x^2 \sqrt{1 - 9x^2}} \, dx$$

5. (11 points) Solve this integral:

$$\int \frac{1}{t^2(1 - t)} \, dt$$

6. (12 points) Find

$$\lim_{n\to\infty} \frac{(2n)!}{n^n}$$

You must justify your answer.

7. (11 points) Does the following series converge or diverge?

$$\sum_{n=1}^{\infty} \frac{1+2n+3n^2}{2+4n^2+6n^3}$$

You must justify your answer.

8. (11 points) Is the following series absolutely convergent, conditionally convergent or divergent?

$$\sum_{n=5}^{\infty} (-1)^n \frac{1}{n \ln n}$$

You must justify your answer.

9. (11 points) Does the following series converge or diverge?

$$\sum_{n=1}^{\infty} \frac{\cos(2n)}{2^n}$$

You must justify your answer.

Part B

10. (13 points) Find a power series representation for

$$\frac{1}{2+x^2}$$

You must show your reasoning. Secondly, find the radius of convergence.

11. (13 points) Let $f(x) = (x-3)^{10}$.

(a) Find the Taylor series for $f(x)$, centered at $a = 3$.

(b) Find $f^{(5)}(3)$

(c) Find $f^{(10)}(3)$

12. (13 points) Suppose we approximate e^x with its Taylor series up to x^3.

$$T_3(x) = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!}$$

Find a number $A > 0$ such that for $x \in [-A, 0]$,

$$|e^x - T_3(x)| \leq 10^{-4}$$

13. (12 points) Find the length of the curve $y = \dfrac{\ln x}{2} - \dfrac{x^2}{4}$, for $1 \leq x \leq 2$.

14. (12 points) Find the area of the surface generated by rotating the curve

$$y = x^3$$

about the x-axis, where x lies between 1 and 3.

15. (12 points) Suppose that a parametric curve is defined by the following equation.

$$\begin{aligned} x(t) &= te^t \\ y(t) &= \sin(t) \end{aligned}$$

Write an integral which represents the length of the curve between $t = 1$ and $t = 2$.

DO NOT SOLVE THE INTEGRAL.

Your integral must be written in terms of the functions $t, e^t, \ln(t)$ and trig functions. It should not explicitly involve $x(t)$ or $y(t)$.

16. (13 points) Find the points on the polar curve $r \sin \theta = 1$ where the tangent line is horizontal.

17. (12 points) Consider the polar curve

$$r = 3\theta^3, \qquad 0 \le \theta \le 2\pi$$

Find the area of the region bounded by the curve and the ray $\theta = 0$, $r \ge 0$.