Math 162: Calculus IIA

Midterm 2

March 24, 2022

NAME (please print legibly): ______________________________
Your University ID Number: ______________________________
Your University email ______________________________
Indicate your instructor with a check in the box:

Sergio Chaves	MW 10:25 - 11:40 AM	
Arda Demirhan	MW 12:30 - 1:45 PM	
Bogdan Krstić	MW 2 - 3:15 PM	
Saul Lubkin	TR 9:40 - 10:55 PM	
Charles Wolf	MW 3:25 - 4:40 PM	

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam and that all work will be my own.

Signature: ______________________________

- **The presence of calculators, cell phones, and other electronic devices at this exam is strictly forbidden. IF YOU HAVE YOUR PHONE WITH YOU, YOU MUST KEEP IT OUT OF REACH OR TURN IT IN TO A PROCTOR BEFORE STARTING THE EXAM. FAILURE TO DO SO WILL BE TREATED AS AN ACADEMIC HONESTY VIOLATION.**
- **Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given. If some of your work is not on the page where the problem appears, indicate where it is.**
- **Put your answers in the space provided at the bottom of each page.**
- **You are responsible for checking that this exam has all 14 pages.**

Handy dandy formulas

Integration by parts formula:

$$\int u\,dv = uv - \int v\,du$$

Trigonometric identities:

$$\cos^2\theta + \sin^2\theta = 1 \qquad \sec^2\theta - \tan^2\theta = 1$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta \qquad \cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\sin 2\theta = 2\sin\theta\cos\theta \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2} \qquad \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

Derivatives of trig functions.

$$\frac{d\sin x}{dx} = \cos x \qquad \frac{d\tan x}{dx} = \sec^2 x \qquad \frac{d\sec x}{dx} = \sec x\tan x$$

$$\frac{d\cos x}{dx} = -\sin x \qquad \frac{d\cot x}{dx} = -\csc^2 x \qquad \frac{d\csc x}{dx} = -\csc x\cot x$$

Nontrivial integrals of trig functions.

$$\int \tan x\,dx = -\log|\cos x| + C = \log|\sec x| + C$$

$$\int \sec x\,dx = \ln|\sec(x) + \tan(x)| + C$$

1. (20 points)

(a) Evaluate the integral $\int_0^{\pi/4} \sec^4(x)\tan^3(x)\,dx$.

ANSWER:

(b) Evaluate the indefinite integral $\int \frac{1}{\sqrt{x^2+2x+2}}\,dx$.

ANSWER:

2. (20 points)

Evaluate

$$\int \frac{x^3 + 2x^2 - x + 2}{x^2\,(x^2+1)}\;dx$$

Hint: Use the method of partial fractions.

ANSWER:

3. (20 points)

For each of the following improper integrals, explain why it is improper, and determine whether it converges or diverges. If it converges, then show to what it converges. Be sure to SHOW ALL WORK.

(a) $\int_0^9 (x-1)^{-1/3} dx$

ANSWER:

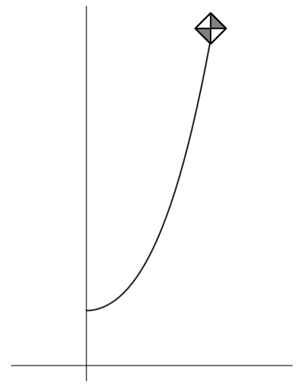
(b) $\displaystyle\int_{-1}^{0} \frac{e^{1/x}}{x^3}\,dx$

ANSWER:

4. (10 points)

The kite's height (in m) above ground with respect to its horizontal position is given by the function

$$y = \frac{1}{6}(x^2 + 4)^{3/2}$$

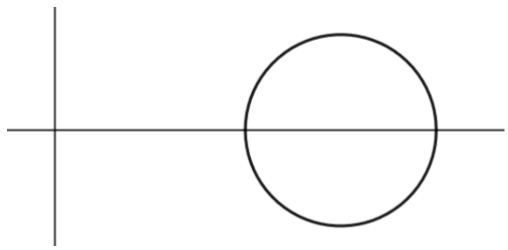


A steady wind blows a kite due west. Find the distance traveled by the kite from $x = 0$ to $x = 3$.

ANSWER:

5. (10 points)

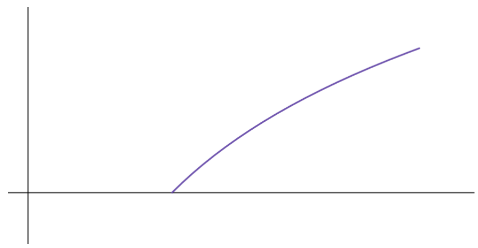
(a) The local bakery makes chocolate dip donuts. The surface of the donut can be described as the surface of revolution obtained by revolving the circumference $(x-3)^2+y^2=1$ about the y-axis. A sketch of the curve is shown below



Assuming that only the upper half of the donut is evenly dipped, write an integral (**do not evaluate it!**) that represents the surface area of the donut covered by chocolate.

ANSWER:

(b) The bakery adds a chocolate dipped strawberry on top of the donut. The strawberry can be described as the surface of revolution obtained by revolving the curve $y = \ln(x)$ about the x-axis from $x = 1$ to $x = \sqrt{e}$. A sketch of the curve is shown below.

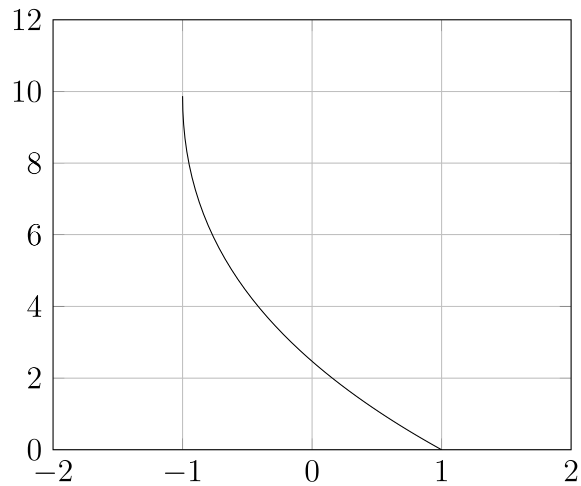


Write an integral (**do not evaluate it!**) that represents the surface area of the strawberry covered by chocolate.

ANSWER:

6. (20 points)

Consider the parametric curve defined by $x(t) = \cos(t)$ and $y(t) = t^2$ for $0 \leq t \leq \pi$.



(a) For $0 \leq t \leq \pi$, find the t values where the tangent line has vertical slope and the t values where the tangent line has horizontal slope.

ANSWER:

(b) Write down **but do not evalute** an integral (with respect to t) expressing the area under the curve from $0 \leq t \leq \pi$.

ANSWER:

Scratch paper

More scratch paper