

Math 162: Calculus IIA

Midterm 1

February 15, 2021

NAME (please print legibly): _____

Your University ID Number: _____

Your University email _____

Indicate your instructor with a check in the box:

Sergio Chaves	MW 10:25 - 11:40 AM	<input type="checkbox"/>
Arda Demirhan	MW 12:30 - 1:45 PM	<input type="checkbox"/>
Bogdan Krstić	MW 2 - 3:15 PM	<input type="checkbox"/>
Saul Lubkin	TR 9:40 - 10:55 PM	<input type="checkbox"/>
Charles Wolf	MW 3:25 - 4:40 PM	<input type="checkbox"/>

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam and that all work will be my own.

Signature: _____

- The presence of calculators, cell phones, and other electronic devices at this exam is strictly forbidden. **IF YOU HAVE YOUR PHONE WITH YOU, YOU MUST KEEP IT OUT OF REACH OR TURN IT IN TO A PROCTOR BEFORE STARTING THE EXAM. FAILURE TO DO SO WILL BE TREATED AS AN ACADEMIC HONESTY VIOLATION.**
- Show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given. If some of your work is not on the page where the problem appears, indicate where it is.
- Put your answers in the space provided at the bottom of each page.
- You are responsible for checking that this exam has all 13 pages.

1. (20 points)

(a) Evaluate the integral $\int_0^{\pi/4} \frac{4x - 6}{1 + x^2} dx$.

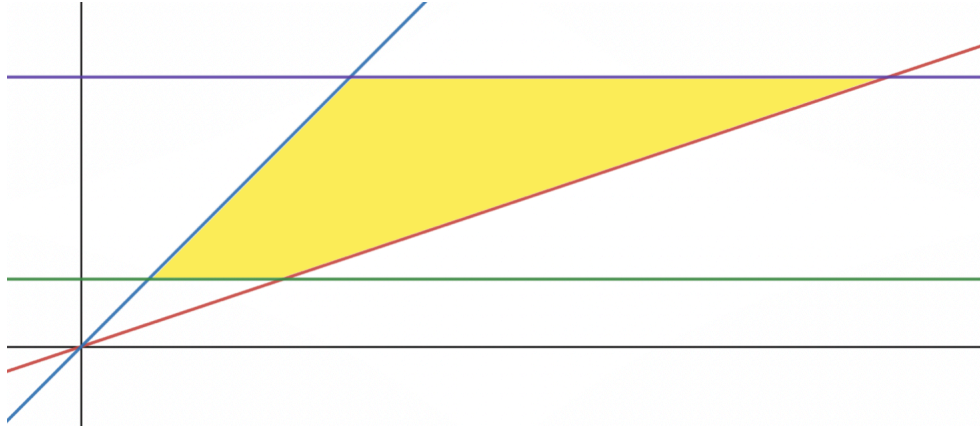
ANSWER:

(b) Evaluate the integral $\int 2x^5 \sqrt{x^2 - 2} dx$.

ANSWER:

2. (20 points)

(a) The lines $y = x$, $y = x/3$, $y = 1$, and $y = 4$ enclose a region in the plane, as shown below:



What is the area of the region enclosed by the lines?

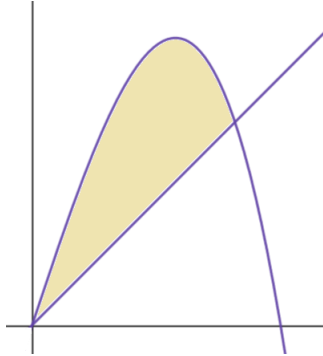
ANSWER:

- (b) Find the value of c such that the horizontal line $y = c$ divides the region from part (a) into two regions of equal area.

ANSWER:

3. (20 points)

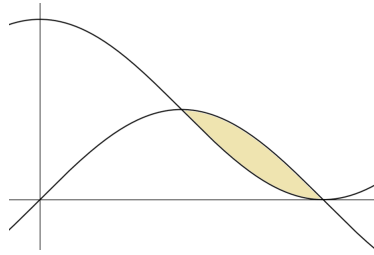
- (a) Consider the region in the first quadrant enclosed by the curve $y = 3x - 2x^3$ and the line $y = x$. A sketch of the region is shown in the following graph:



Write down, **but do not evaluate** an integral expressing the volume of the solid obtained by revolving the region about the y -axis.

ANSWER:

- (b) Consider the region enclosed by the curves $y = \sin(x)$ and $y = 1 + \cos(x)$, and the lines $x = \pi/2$ and $x = \pi$. A sketch of the region is shown in the following graph.



Write down, **but do not evaluate** an integral expressing the volume of the solid obtained by revolving the region about the line $y = 1$.

ANSWER:

4. (20 points)

A pool has a radius of 5 meters and a height of 4 meters. It is filled halfway with an unknown liquid. This liquid is then fully drained from the top of the pool, which takes $1,800,000\pi$ Joules of energy. What is the density of the liquid? [Approximate gravity as 10 m/s^2 .]

ANSWER:

5. (20 points)

(a) Use Integration by Parts to show that, for every integer $n \geq 1$,

$$\int (\ln x)^n dx = x(\ln x)^n - n \int (\ln x)^{n-1} dx.$$

(b) Use (a) to compute $\int \ln x \, dx$.

ANSWER:

(c) Use (a) and (b) to compute $\int (\ln x)^2 dx$.

ANSWER:

Scratch paper

More scratch paper