MTH161 Workshop 9: The Mean Value Theorem; Curve Sketching

Discussion Questions: With your group, discuss the Mean Value Theorem in your own words. What conditions does a function have to satisfy in order for the theorem to apply?

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1. Consider the following problems with your group.

(a) Let $f(x) = 5 - x^{2/3}$. Show that $f(1) = f(-1)$, but there is no number c in the open interval $(-1, 1)$ such that $f'(c) = 0$. Why does this not contradict Rolle's Theorem (or the Mean Value Theorem)?

(b) Two runners start a race at the same time and finish in a tie. With your group, prove that at some time during the race they have the same speed. (**Hint:** Let $f(t)$ and $g(t)$ be the position functions of the two runners, and consider the function $h(t) = f(t) - g(t)$.)

2. Consider the following problems with your group.

(a) With your group, sketch the graphs of three different functions f that satisfy the following properties:
- f is continuous on $[1, 5]$;
- f has an absolute maximum at 5;
- f has a local maximum at 2;
- f has a local minimum at 4.

(b) Suppose that $f'(6) = 0$ and $f''(6) = 1$. Determine with your group what the graph of f must look like ***near*** $x = 6$. Sketch what this portion of the graph looks like.

3. Sketch a graph of a function $f(x)$ that satisfies all of the given conditions:
(a) $f(-2) = f(2) = f(6) = 0$.
(b) $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = 3$.
(c) $\lim_{x\to1} f(x) = \infty$.
(d) $f'(4) = 0$.
(e) $f'(x) > 0$ if $x < 1$ or $x > 4$, and $f'(x) < 0$ if $1 < x < 4$.
(f) $f''(x) > 0$ if $x < 1$ or $1 < x < 7$, and $f''(x) < 0$ if $x > 7$.

4. Let $f(x) = \dfrac{1+x}{x^{2/3}}$. Note that Complete the following with your group, **without the aid of a calculator**:

(a) Find the domain of f.
(b) Find all x- and y-intercepts.
(c) Find all vertical asymptotes.
(d) Find the limits at $\pm\infty$ and any horizontal asymptotes.
(e) Where is $f(x)$ increasing? decreasing? Where does f have a local maximum/minimum?
(f) Where is $f(x)$ concave up? concave down? Where does f have an inflection point?
(g) Use all of this information to sketch a graph of f.

5. Let $f(x) = \dfrac{\ln x}{x}$. Note that

$$f'(x) = \frac{1-\ln x}{x^2} \text{ and } f''(x) = \frac{2\ln x - 3}{x^3}.$$

Complete the following with your group, **without the aid of a calculator**:

(a) Find the domain of f.
(b) Find all x- and y-intercepts.
(c) Find all vertical asymptotes.
(d) Where is $f(x)$ increasing? decreasing? Where does f have a local maximum/minimum?
(e) Where is $f(x)$ concave up? concave down? Where does f have an inflection point?
(f) What is the behavior of f "near infinity?"
(g) Use all of this information to sketch a graph of f.