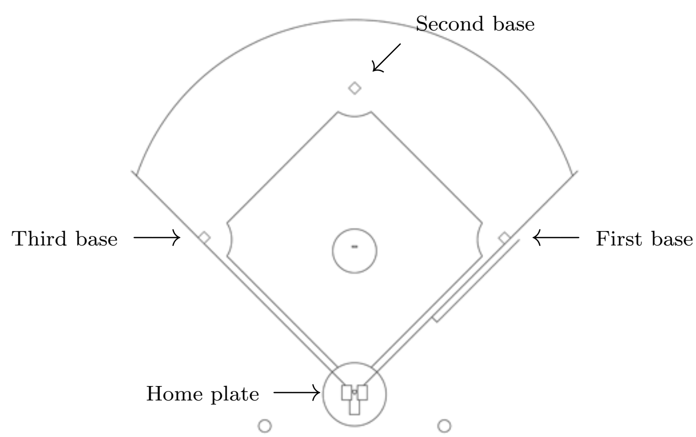
MTH161 Workshop 8: related rates; linearization; max/min values

1. A baseball diamond is a square with side length 90 ft. Jason hits the ball and runs from home plate toward first base at a speed of 26 ft/s. At the moment he is 30 ft from first base,

(a) at what rate is his distance from second base decreasing?

(b) at what rate is his distance from third base increasing?



2. A Ferris wheel with a radius of 10 m is rotating at a rate of one revolution every 2 minutes. With your group, determine how fast a rider is *rising vertically* when his seat is 16 m above ground level. (Remember, draw a picture first!)

3. Consider the following functions:

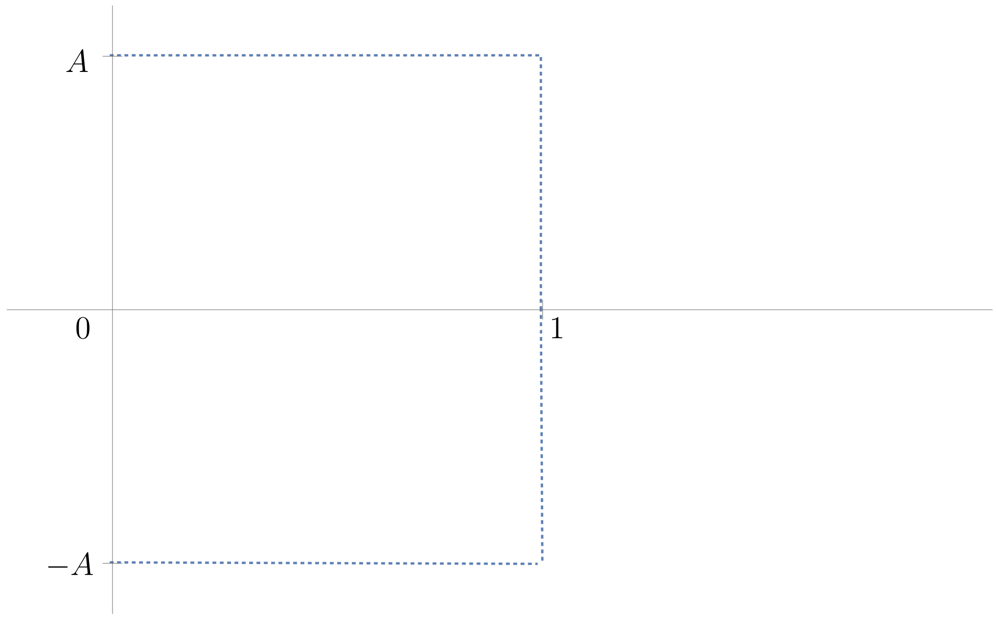
$$f(x) = (x-1)^2$$
$$g(x) = e^{-2x}$$
$$h(x) = 1 + \ln(1-2x).$$

(a) Find the linearizations of f, g, and h at $a = 0$. What do you notice?

(b) As a group, sketch the graphs of f, g, and h, and their linear approximations. For which of these three functions does the linear approximation seem best? For which does it seem worst?

4. Determine whether the following statement is true or false:

Let f be any continuous function with domain $[0,1]$. Then there exists a positive number A such that the graph of f can fit inside the rectangle consisting of all points (x,y) in the plane with $0 \le x \le 1$ and $-A \le y \le A$.



How would your answer change if we no longer required f to be continuous? What about if f is continuous but we replace the closed interval $[0,1]$ with the open interval $(0,1)$?

5. Determine with your group the values of a and b for which the function

$$f(x) = x^3 + ax^2 + bx + 2$$

has a local maximum at $x = -3$ and a local minimum at $x = -1$.

6. Consider the following function:

$$f(x) = x^a(1-x)^b,$$

where a and b are positive constants. Show that the maximum value of $f(x)$ on the interval $[0,1]$ is $\dfrac{a^a b^b}{(a+b)^{a+b}}$.

Challenge Problem: A plane flying at a constant speed of 300 km/hr passes over a ground radar station at an altitude of 1km and climbs at an angle of 30°. At what rate is the distance from the plane to the radar station increasing one minute later?

(**Hint:** You'll need the law of cosines, which says that if a, b, and c are the sides of a triangle, and θ is the angle opposite the side of length c, then $c^2 = a^2 + b^2 - 2ab\cos\theta$.)