

Worksheet 7

Implicit Differentiation, Derivatives of Logs, Logarithmic Differentiation, Rates of Change and 1-D Motion, Related Rates

Problem Set Instructions: Work through the following problems with your group. You might not finish all of the problems, but be sure to work on all of them together and gain a good idea of how to proceed.

1. Complete the following:

(a) Show that $\csc^{-1}(x) = \sin^{-1}\left(\frac{1}{x}\right)$.

(b) Use part (a) to show $\frac{d}{dx} \csc^{-1}(x) = \frac{-1}{x^2 \sqrt{1 - \frac{1}{x^2}}} = \frac{-1}{|x| \sqrt{x^2 - 1}}$.

(c) Similarly write $\sec^{-1}(x)$ in terms of $\cos^{-1}(x)$ and find the derivative of $\sec^{-1}(x)$.

2. Let h , k , and r be constants, and consider the equations

$$(x - h)^2 + (y - k)^2 = r^2$$

(a) Sketch the graph of the equations. Label your graph. In particular, show how the constants h , k , and r are relevant to the graph.

(b) By just looking at the graph, determine at which points the tangent line is horizontal and at which points the tangent line is vertical.

(c) Now, find $\frac{dy}{dx}$ and verify your answer to part (b).

3. Consider

$$\frac{e^x \sqrt{x}(x-2)^3}{\sqrt[4]{(x^2+1)^3} \ln(x)}$$

(a) Use Logarithmic differentiation to find $f'(x)$.

(b) Find $f'(2)$.

4. Consider the equation $x^y = y^x$. Find $\frac{dy}{dx}$ in terms of x and y .

5. An object is traveling along the x -axis. Its position at time t seconds ($t \geq 0$) is given by

$$x(t) = t^3 - 9t^2 + 24t \text{ ft.}$$

With your group, answer the following questions about the motion of the object.

- (a) What is the velocity at time t ?
- (b) When is the object at rest?
- (c) When is the particle moving in the forward direction? In the backward direction?
- (d) What is the acceleration at time t ?
- (e) when is the particle speeding up? Slowing down?
(**Hint:** The object is speeding up when it's accelerating in the same direction it's moving and slowing down when it's accelerating in the opposite direction.)
- (f) What is the total distance the object travels during the first 8 seconds?

6. A Ferris Wheel with a radius of 10m is rotating at a rate of one revolution every 2 minutes. With your group, determine how fast a rider is rising vertically when his seat is 16m above ground level. (Remember, draw a picture first!)