MTH161 Workshop 6: product rule, quotient rule, chain rule

1. Let $f(x) = \sec x$.
(a) Use the quotient rule to show $f'(x) = \sec x \tan x$.
(b) Use the chain rule to show $f'(x) = \sec x \tan x$.

2. (a) Find an example to show that the derivative of $f(x)g(x)$ is not equal to $f'(x)g'(x)$.
(b) Show that if f, g, and h are differentiable functions, then $(fgh)' = f'gh + fg'h + fgh'$. (Hint: Use the Product Rule)
(c) Find $\frac{d}{dx}(4x^3\sqrt{x}e^x \sin(x))$.

3. Find

$$\frac{d}{dx} e^{e^{e^{e^{e^x}}}}$$

4. An object with weight W is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle θ with the plane, then the magnitude of the force is

$$F = \frac{\mu W}{\mu \sin\theta + \cos\theta},$$

where μ is a constant called the *coefficient of friction*.
(a) Find the rate of change of F with respect to θ.
(b) When is this rate of change equal to 0?

5. Recall that a function $f(x)$ is **even** if $f(-x) = f(x)$ and **odd** if $f(-x) = -f(x)$.
(a) If $f(x)$ is an even function, is $f'(x)$ even, odd, or neither?
(b) If $f(x)$ is an odd function, is $f'(x)$ even, odd, or neither?

6. Let r be a constant. For what value(s) of r does the function $y = e^{rx}$ satisfy the differential equation $y'' - 4y' + y = 0$?

7. Consider the function $f(x) = 2\sin x + \sin^2 x$. Below are two solutions to find all points on the graph such that the tangent line is horizontal. Is either of them correct? With your group, (1) pick the correct solution (if any); (2) point out the error(s) in the incorrect solution(s); and (3) write a correct solution if neither solution is correct.

(a) First, find $f'(x)$:

$$f'(x) = 2\cos x + 2\sin x.$$

Now, set $f'(x) = 0$ and solve for x:

$$\begin{aligned} f'(x) &= 0 \\ 2\cos x + 2\sin x &= 0 \\ 2\sin x &= -2\cos x \\ \tan x &= -1 \\ x &= \boxed{\frac{3\pi}{4} + \pi k, \text{ where } k \text{ is any integer}} \end{aligned}$$

(b) First, find $f'(x)$:

$$f'(x) = 2\cos x + 2\sin x\cos x.$$

Now, set $f'(x) = 0$ and solve for x:

$$\begin{aligned} f'(x) &= 0 \\ 2\cos x + 2\sin x\cos x &= 0 \\ 2\sin x\cos x &= -2\cos x \\ 2\sin x &= -2 \\ \sin x &= -1 \\ x &= \boxed{\frac{3\pi}{2} + 2\pi k, \text{ where } k \text{ is any integer}} \end{aligned}$$