

## Worksheet 6

### Chain Rule

**Problem Set Instructions:** Work through the following problems with your group. You might not finish all of the problems, but be sure to work on all of them together and gain a good idea of how to proceed.

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1. Let  $f(x) = \sec(x)$ . Use the chain rule to show that  $f'(x) = \sec(x) \tan(x)$ .

2. Find

$$\frac{d}{dx} e^{e^{e^{e^{e^x}}}}$$

3. Recall that a function  $f(x)$  is **even** if  $f(-x) = f(x)$  and **odd** if  $f(-x) = -f(x)$ .

- (a) If  $f(x)$  is an even function, is  $f'(x)$  even, odd, or neither.
- (b) If  $f(x)$  is an odd function, is  $f'(x)$  even, odd, or neither.

4. Recall that  $f(f^{-1}(x)) = x$  for every  $x$  where  $f^{-1}$  is well defined.

- (a) Use the above property for inverse function to compute  $\frac{d}{dx} f^{-1}(x)$  in terms of  $f'$  and  $f^{-1}$ .
- (b) Use part (a) to find an expression for

$$\frac{d}{dx} \tan^{-1}(x)$$

that does not contain any trig functions. Hint: The answer should be a rational function.

- (c) Now repeat part (b) but with  $\frac{d}{dx} \sin^{-1}(x)$  and  $\frac{d}{dx} \cos^{-1}(x)$ .

5. Let  $r$  be a constant. For what value(s) of  $r$  does the function  $y = e^{rx}$  satisfy the differential equation  $y'' + 4y' + y = 0$ ?

6. Under certain circumstances a rumor spreads according to the equation

$$p(t) = \frac{1}{1 + ae^{-kt}}$$

where  $p(t)$  is the proportion of the population that has heard the rumor at time  $t$  and  $a$  and  $k$  are positive constants.

- (a) When will half of the population have heard the rumor? happens to  $p(t)$  as  $t \rightarrow \infty$ ? What does this mean?
- (b) Find  $p'(t)$ . What does this represent?
- (c) What happens to  $p'(t)$  as  $t \rightarrow \infty$ ? What does this mean?