

Worksheet 5

Product Rule, Quotient Rule, and Derivatives of Trigonometric Functions

Problem Set Instructions: Work through the following problems with your group. You might not finish all of the problems, but be sure to work on all of them together and gain a good idea of how to proceed.

- Let $f(x) = \sec(x)$, $g(x) = \csc(x)$, and $h(x) = \cot(x)$.
 - Use the quotient rule to show that $f'(x) = \sec(x) \tan(x)$.
 - Use the quotient rule to show that $g'(x) = -\csc(x) \cot(x)$.
 - Use the quotient rule to show that $h'(x) = -\csc^2(x)$.

- Find an example to show that the derivative of $f(x)g(x) \neq f'(x)g'(x)$.
 - Show that if f, g , and h are differentiable functions, then $(fgh)' = f'gh + fg'h + fgh'$.
(Hint: Use the Product Rule)
 - Find $\frac{d}{dx}(4x^3\sqrt{x}e^x \sin(x))$

- An object with weight W is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle θ with the plane, then the magnitude of the force is

$$\frac{\mu W}{\mu \sin(\theta) + \cos(\theta)}$$

Where μ is a constant called the coefficient of friction.

- Find the rate of change of F with respect to θ .
- When is the rate of change equal to 0?

4. Let $f(x) = \frac{x - x^2}{1 + 8x^2}$. Determine where $f(x)$ is increasing and where it is decreasing.

5. Consider the function $f(x) = 2 \sin(x) + \sin^2(x)$. Below are two solutions to find all points on the graph such that the tangent line is horizontal. Is either of them correct? With your group, (1) pick the correct solution (if any); (2) point out the error(s) in the incorrect solution(s); and (3) write a correct solution if neither solution is correct.

(a) First, find $f'(x)$:

$$f'(x) = 2 \cos(x) + 2 \sin(x).$$

Now, set $f'(x) = 0$ and solve for x :

$$\begin{aligned} f'(x) &= 0 \\ 2 \cos(x) + 2 \sin(x) &= 0 \\ 2 \sin(x) &= -2 \cos(x) \\ \tan(x) &= -1 \\ x &= \frac{3\pi}{4} + \pi k, \text{ where } k \text{ is any integer.} \end{aligned}$$

(b) First, find $f'(x)$:

$$f'(x) = 2 \cos(x) + 2 \sin(x) \cos(x).$$

Now, set $f'(x) = 0$ and solve for x :

$$\begin{aligned} f'(x) &= 0 \\ 2 \cos(x) + 2 \sin(x) \cos(x) &= 0 \\ 2 \sin(x) \cos(x) &= -2 \cos(x) \\ 2 \sin(x) &= -2 \\ \sin(x) &= -1 \\ x &= \frac{3\pi}{2} + 2\pi k, \text{ where } k \text{ is any integer.} \end{aligned}$$