MTH161 Workshop 3: tangents, velocity, and limits.

Discussion:

- Discuss with your group how slopes of secant lines are used to approximate slopes of tangent lines.
- Discuss with your group, in your own words, what the equation $\lim_{x\to a} f(x) = L$ means.

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1. The table below shows the position of a cyclist s meters from her starting point after t seconds.

t	0	1	2	3	4	5
s	0	1.4	5.1	10.7	17.7	25.8

(a) Find the average velocity of the cyclist over each time period below.

(i) $[1, 3]$
(ii) $[2, 3]$
(iii) $[3, 5]$
(iv) $[3, 4]$

(b) Plot the points above to sketch the graph of $s(t)$. Referring to the graph, what do the average velocities from part (a) represent?

(c) Use your sketch to *estimate* the instantaneous velocity when $t = 3$.

2. (a) Consider the quantity

$$\frac{\dfrac{1}{(4+h)^2} - \dfrac{1}{16}}{h}.$$

This can be viewed as the slope of a secant line through the points $(4, f(4))$ and $(4 + h, f(4 + h))$ on the graph of a certain function. What is the function?

(b) Make a sketch of the function you found in part (a) with the secant line and points labeled.

(c) Now evaluate the limit:

$$\lim_{h\to 0} \frac{\dfrac{1}{(4+h)^2} - \dfrac{1}{16}}{h}.$$

Hint: as a first step, simplify $\dfrac{1}{(4+h)^2} - \dfrac{1}{16}$ into a single fraction $\dfrac{A}{B}$.

(d) The limit can be viewed as the slope of the tangent line to the graph of a certain function at a certain point. What function and what point? Add the tangent line to your sketch from part (b).

3. With your group, sketch the graph of a single function with all of the following properties:

(a) $\lim_{x\to 0^-} f(x) = 2$.
(b) $\lim_{x\to 0^+} f(x) = 0$.
(c) $\lim_{x\to 4^-} f(x) = 2$.
(d) $\lim_{x\to 4^+} f(x) = 0$.
(e) $x = 2$ is a vertical asymptote for f.
(f) $f(0) = 2$ and $f(4) = 1$.

Can you draw a second graph with the same properties?

4. The *signum* function, denoted sgn, is defined by

$$\operatorname{sgn} x = \begin{cases} -1, & \text{if } x < 0; \\ 0, & \text{if } x = 0; \\ 1, & \text{if } x > 0. \end{cases}$$

(a) With your group, sketch the graph of this function.
(b) Find each of the following limits, or explain why the limit doesn't exist.

(i) $\lim_{x\to 0^+} \operatorname{sgn} x$
(ii) $\lim_{x\to 0^-} \operatorname{sgn} x$
(iii) $\lim_{x\to 0} \operatorname{sgn} x$
(iv) $\lim_{x\to 0} |\operatorname{sgn} x|$

5. In the theory of relativity, the mass of a particle with velocity v is given by

$$m = \frac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}},$$

where m_0 is the mass of the particle at rest and c is the speed of light. What happens to the mass as v approaches c from below?