MTH161 Workshop 11: Anti-derivatives; Area and Definite Integrals

Discussion Questions: Discuss the following questions with your group.

- What is the procedure for **approximating** the area under the curve $y = f(x) \geq 0$ between $x = a$ and $x = b$?
- What does the notation $\sum_{i=1}^{n} a_i$ mean?

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1. Write the anti-derivatives to the following functions without the aide of notes or textbook (make sure you can before the final exam!!):

x^n, where $n \neq -1$	$\dfrac{1}{x}$
e^x	a^x, where $a > 0$
$\sin x$	$\cos x$
$\sec^2 x$	$\sec x \tan x$
$\csc^2 x$	$\csc x \cot x$
$\dfrac{1}{1+x^2}$	$\dfrac{1}{\sqrt{1-x^2}}$

2. Consider the following antiderivative problems:

(a) Given that the graph of f passes through the point $(1, -4)$ and that the slope of the tangent line at any point $(x, f(x))$ is $2x + 1$, find the x-intercepts of f.

(b) A particle is moving so that its acceleration at time t is given by $a(t) = \alpha \cos t - \beta \sin t$, where α and β are constants. Its position at time $t = 0$ is $s(0) = 0$, and its velocity at time $t = \pi/2$ is equal to 4. Find the position of the particle at time t. (Your answer may include α and β.)

3. A car is traveling at 30 m/s when the driver sees a roadblock 80 m ahead and slams on the brakes. What constant deceleration is required to stop the car in time to avoid an accident?

4. You want to estimate the area under the graph of a positive function by using four rectangles of equal width. The rectangles that must give the best estimate of this area are those with height obtained from the

(a) left endpoints
(b) midpoints
(c) right endpoints
(d) there's not enough information to decide

5. With your group, approximate the area under the curve $y = \sqrt{x}$ between $x = 3$ and $x = 12$ using a **right endpoint** approximation with **three** rectangles of equal width.

6. Consider the following integral with your group:

$$\int_0^2 \left(\sqrt{4 - x^2} + 3x\right)\, dx$$

(a) Estimate the integral by using four rectangles of equal width and right endpoints.
(b) Write the integral as the limit of a Riemann sum using right endpoints. (Don't try to *evaluate* the limit — just write down the appropriate limit.)
(c) Evaluate the definite integral by interpreting it in terms of areas. (**Hint:** Start by separating the integral so that you can integrate $\sqrt{4 - x^2}$ and $3x$ separately.)

7. Hank was trying to compute a definite integral using the limit definition, but he got distracted by the beautiful weather and went outside, leaving his work on his desk. You find the following limit written on his paper:

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{2 + \frac{3i}{n}} \cdot \left(\frac{3}{n}\right).$$

(a) What integral was Hank trying to compute? In other words, on which interval $[a, b]$ and for which function $f(x)$ does the above limit equal

$$\int_a^b f(x)\, dx \ ?$$

(b) Can you find a different interval and function that works?
(c) If you are told the interval is $[-1, 5]$, what must the function f be?