MTH161 Workshop 10: L'Hospital's Rule; applied optimization

Discussion Questions:

Does L'Hôpital's rule work **any time** you want to take the limit of $\dfrac{f(x)}{g(x)}$?

1. (a) Show that $\lim_{x\to\infty} \dfrac{x^n}{e^x} = 0$ (or, equivalently, $\lim_{x\to\infty} \dfrac{e^x}{x^n} = \infty$) for all positive numbers n, and discuss what this means about the growth of the exponential function e^x.

(b) Explain to each other why $\lim_{x\to\infty} \dfrac{\ln x}{x^p} = 0$ for any positive number p. What does this mean about the growth of $\ln(x)$?

2. (a) Show that

$$\lim_{x\to\infty} (1 + a/x)^{x/b} = e^{a/b}.$$

(b) Explain why part (a) implies

$$\lim_{x\to 0} (1 + ax)^{1/(bx)} = e^{a/b}.$$

Hint: try a change of variable in the limit, $t = 1/x$.

3. A pizza parlor has the following special: For \$2, you can order a slice of pizza as large as you want, as long as the perimeter of the slice is no more than 24 inches. (A slice of pizza is in the shape of a sector of a circle.) What diameter should the pizza have in order to give you the largest slice? (Here "largest" means "largest area.")

4. Hank has a piece of cardboard that is 5 in. by 8 in. He needs to construct an open-top box by cutting a square from each corner and folding up the sides. (**Note:** The square cut from each corner is the same size for all four corners.) What is the largest possible volume of such a box?

5. What is the area of the largest rectangle that can be inscribed in a circle of radius r? (Your answer will be in terms of r.)

6. A Norman window has the shape of a rectangle with a semicircle on top. (The diameter of the semicircle is equal to the width of the rectangle.) If the perimeter of the window must be 30 ft, find the dimensions of the window with the greatest possible area.

Challenge problem: The figure below shows a sector of a circle with central angle θ. Let $A(\theta)$ be the area of the segment between the chord PQ and the arc PQ. Let $B(\theta)$ be the area of the triangle PQR. Find $\lim_{\theta \to 0^+} \frac{A(\theta)}{B(\theta)}$.