MTH161 Workshop 1: inequalities, lines, and trigonometry

Discussion: Discuss the following question with your group.
Consider the statement

$$\sqrt{x^2} = x \text{ and } (\sqrt{x})^2 = x.$$

Is this true for all real numbers x? If so, explain why. If not, fix the statement to make it true for all real numbers x.

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1. We want to solve the following inequality for x:

$$x^2 > 4x.$$

(a) Discuss with your group what is wrong with the following "solution":

$$\begin{aligned} x^2 &> 4x \\ x &> \sqrt{4x} \\ x &> 2\sqrt{x} \\ \sqrt{x} &> 2 \\ x &> 4 \end{aligned}$$

Answer: $(4, \infty)$

(b) Discuss with your group what is wrong with the following "solution":

$$\begin{aligned} x^2 &> 4x \\ x^2 - 4x &> 0 \\ x(x-4) &> 0 \\ x > 0 &\text{ or } x - 4 > 0 \end{aligned}$$

Answer: $(0, \infty) \cup (4, \infty)$

(c) Now find the correct solution.

2. (a) Show that if the x- and y-intercepts of a line are nonzero numbers a and b, respectively, then the equation of the line can be written in the form

$$\frac{x}{a} + \frac{y}{b} = 1.$$

This is called the **two-intercept form** of an equation of a line.

(b) Use part (a) to find an equation of the line whose x-intercept is 6 and whose y-intercept is -8.

3. Show that the points $(2, 1)$, $(8, 4)$, $(6, 8)$, and $(0, 5)$ are vertices of a parallelogram. Is this parallelogram a rectangle? A square?

Hint: First, help your scribe draw a picture in the coordinate plane. Discuss with your group what defines a parallelogram, rectangle, and square, then proceed.

4. Let $\theta = \dfrac{5\pi}{7}$ (radians). You would need a calculator to find the exact trigonometric values for θ. You won't need that here.

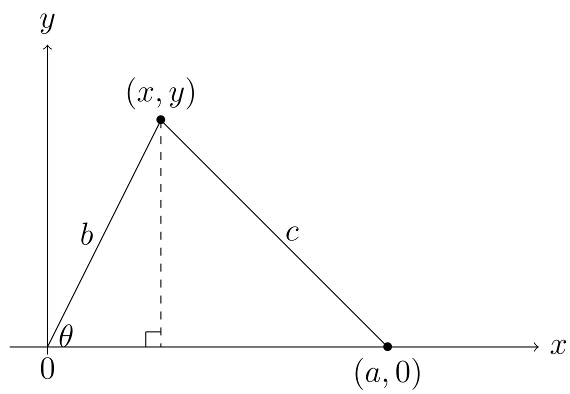
(a) Help your scribe draw the angle θ in standard position in the plane. In what quadrant does it lie?

(b) Can you find an angle α with $0 \le \alpha \le 2\pi$ with exactly **two** of the same trigonometric values as θ? (For example, is it possible to find an angle α with $\sin(\alpha) = \sin(\theta)$ and $\cot(\alpha) = \cot(\theta)$, but with no other shared trigonometric values?) If so, find such an α. If not, explain. If such an α exists, draw it in standard position together with θ.

(c) Can you find an angle γ with $0 \le \gamma \le 2\pi$ that has all **six** of the same trigonometric values as θ? If so, find such a γ. If not, explain. What if you expanded the possible range for γ to be $[0, 4\pi]$? What about $[-2\pi, 2\pi]$? If such a γ exists, draw it in standard position together with θ.

5. Law of Cosines: If a triangle has sides with lengths $a, b,$ and c, and θ is the angle between the sides with lengths a and b, then

$$c^2 = a^2 + b^2 - 2ab\cos\theta.$$



With your group, prove the Law of Cosines assuming $0 < \theta < \pi/2$. **Hint**: Use two different triangles to write y in two different ways in terms of $a, b, c,$ and x.

6. (a) Use Pythagorean identity and the cosine double angle formula

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

to write $\cos^2(x)$ in terms of $\cos(2x)$ but no other trig functions. Similarly write $\sin^2(x)$ in terms of $\cos(2x)$ but no other trig functions.

(b) The sine, cosine addition formulas are given by

$$\begin{aligned} \sin(x \pm y) &= \sin(x)\cos(y) \pm \cos(x)\sin(y) \\ \cos(x \pm y) &= \cos(x)\cos(y) \mp \sin(x)\sin(y) \end{aligned}$$

Use these to write $\sin(x)\sin(y), \cos(x)\cos(y)$, and $\sin(x)\cos(y)$ in terms of $\sin(x+y), \sin(x-y), \cos(x+y)$, and $\cos(x-y)$.

Challenge problem: What if we assume $\pi/2 < \theta < \pi$ in problem 5? How does this change your solution?