MATH161 Workshop 8: Curve Sketching; L'Hospital's Rule

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1. Sketch a graph of a function $f(x)$ that satisfies all of the given conditions:

(a) $f(-2) = f(2) = f(6) = 0$.

(b) $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = 3$.

(c) $\lim_{x\to1} f(x) = \infty$.

(d) $f'(4) = 0$.

(e) $f'(x) > 0$ if $x < 1$ or $x > 4$, and $f'(x) < 0$ if $1 < x < 4$.

(f) $f''(x) > 0$ if $x < 1$ or $1 < x < 7$, and $f''(x) < 0$ if $x > 7$.

2. Let $f(x) = \dfrac{1+x}{x^{2/3}}$. Note that Complete the following with your group, **without the aid of a calculator**:

(a) Find the domain of f.

(b) Find all x- and y-intercepts.

(c) Find all vertical asymptotes.

(d) Find the limits at $\pm\infty$ and any horizontal asymptotes.

(e) Where is $f(x)$ increasing? decreasing? Where does f have a local maximum/minimum?

(f) Where is $f(x)$ concave up? concave down? Where does f have an inflection point?

(g) Use all of this information to sketch a graph of f.

3. Let $f(x) = \dfrac{\ln x}{x}$. Note that

$$f'(x) = \frac{1-\ln x}{x^2} \text{ and } f''(x) = \frac{2\ln x - 3}{x^3}.$$

Complete the following with your group, **without the aid of a calculator**:

(a) Find the domain of f.

(b) Find all x- and y-intercepts.

(c) Find all vertical asymptotes.

(d) Where is $f(x)$ increasing? decreasing? Where does f have a local maximum/minimum?

(e) Where is $f(x)$ concave up? concave down? Where does f have an inflection point?

(f) What is the behavior of f "near infinity?"

(g) Use all of this information to sketch a graph of f.

4. Let $f(x) = xe^{-x^2}$. Note that Complete the following with your group, **without the aid of a calculator**:

(a) Find the domain of f.
(b) Find all x- and y-intercepts.
(c) Find all vertical asymptotes.
(d) Find the limits at $\pm\infty$ and any horizontal asymptotes.
(e) Where is $f(x)$ increasing? decreasing? Where does f have a local maximum/minimum?
(f) Where is $f(x)$ concave up? concave down? Where does f have an inflection point?
(g) Use all of this information to sketch a graph of f.

5. (a) Show that $\lim_{x\to\infty} \frac{x^n}{e^x} = 0$ (or, equivalently, $\lim_{x\to\infty} \frac{e^x}{x^n} = \infty$) for all positive numbers n, and discuss what this means about the growth of the exponential function e^x.

(b) Explain to each other why $\lim_{x\to\infty} \frac{\ln x}{x^p} = 0$ for any positive number p. What does this mean about the growth of $\ln(x)$?

Challenge problem: The figure below shows a sector of a circle with central angle θ. Let $A(\theta)$ be the area of the segment between the chord PQ and the arc PQ. Let $B(\theta)$ be the area of the triangle PQR. Find $\lim_{\theta\to 0^+} \frac{A(\theta)}{B(\theta)}$.