MATH161 Workshop 7: Linear Approximation; Max/Min Values; MVT

Discussion Questions: With your group, discuss the Mean Value Theorem in your own words. What conditions does a function have to satisfy in order for the theorem to apply?

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1. Consider the following functions:

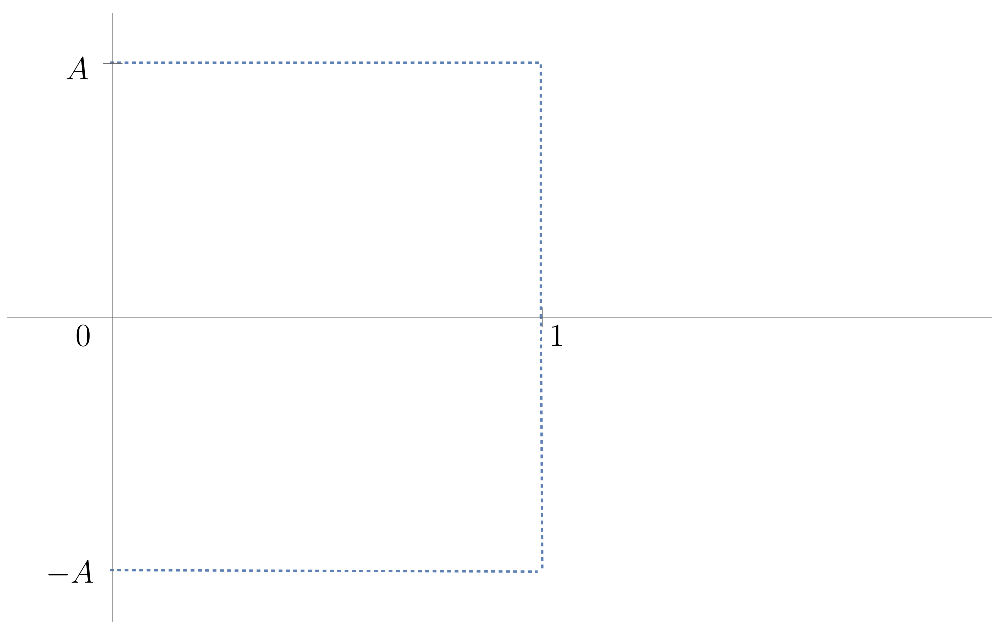
$$f(x) = (x-1)^2$$
$$g(x) = e^{-2x}$$
$$h(x) = 1 + \ln(1-2x).$$

(a) Find the linearizations of f, g, and h at $a = 0$. What do you notice?

(b) As a group, sketch the graphs of f, g, and h, and their linear approximations. For which of these three functions does the linear approximation seem best? For which does it seem worst?

2. Determine whether the following statement is true or false:

Let f be any continuous function with domain $[0,1]$. Then there exists a positive number A such that the graph of f can fit inside the rectangle consisting of all points (x,y) in the plane with $0 \leq x \leq 1$ and $-A \leq y \leq A$.



How would your answer change if we no longer required f to be continuous? What about if f is continuous but we replace the closed interval $[0,1]$ with the open interval $(0,1)$?

3. Determine with your group the values of a and b for which the function

$$f(x) = x^3 + ax^2 + bx + 2$$

has a local maximum at $x = -3$ and a local minimum at $x = -1$.

4. Consider the following function:

$$f(x) = x^a(1-x)^b,$$

where a and b are positive constants. Show that the maximum value of $f(x)$ on the interval $[0, 1]$ is $\dfrac{a^a b^b}{(a+b)^{a+b}}$.

5. Consider the following problems with your group.

(a) Let $f(x) = 5 - x^{2/3}$. Show that $f(1) = f(-1)$, but there is no number c in the open interval $(-1, 1)$ such that $f'(c) = 0$. Why does this not contradict Rolle's Theorem (or the Mean Value Theorem)?

(b) Two runners start a race at the same time and finish in a tie. With your group, prove that at some time during the race they have the same speed. (**Hint:** Let $f(t)$ and $g(t)$ be the position functions of the two runners, and consider the function $h(t) = f(t) - g(t)$.)

6. Consider the following problems with your group.

(a) With your group, sketch the graphs of three different functions f that satisfy the following properties:

- f is continuous on $[1, 5]$;
- f has an absolute maximum at 5;
- f has a local maximum at 2;
- f has a local minimum at 4.

(b) Suppose that $f'(6) = 0$ and $f''(6) = 1$. Determine with your group what the graph of f must look like ***near*** $x = 6$. Sketch what this portion of the graph looks like.