**MTH161 Workshop 6: Logarithmic Differentiation; Rates of Change; Related Rates; Linearization**

**Problem Set Instructions:** Work through the following problems with your group. You might not finish all of the problems, but be sure to work on all of them together and gain a good idea of how to proceed.

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**1.** Consider

$$f(x) = \frac{e^x \sqrt{x}(x-2)^3}{\sqrt[4]{(x^2+1)^3}\ln(x)}$$

(a) Use logarithmic differentiation to find $f'(x)$.

(b) Find $f'(2)$.

**2.** Consider the equation $x^y = y^x$. Find $\dfrac{dy}{dx}$ in terms of $x$ and $y$.

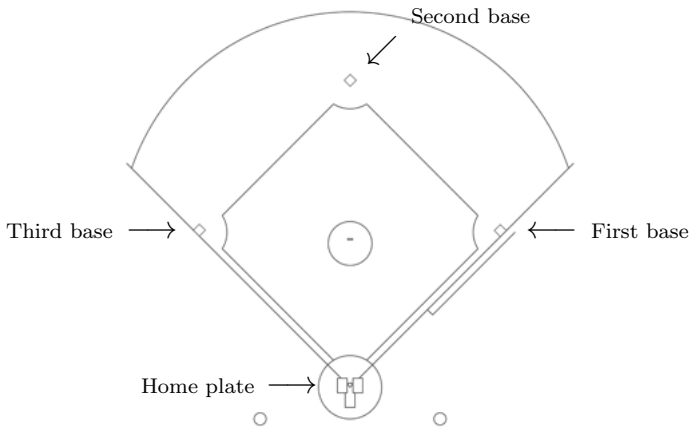
**3.** An object is traveling along the $x$-axis. Its position at time $t$ seconds ($t \geq 0$) is given by

$$x(t) = t^3 - 9t^2 + 24t \text{ ft.}$$

With your group, answer the following questions about the motion of the object.

(a) What is the velocity at time $t$?
(b) When is the object at rest?
(c) When is the particle moving in the *forward* direction? In the *backward* direction?
(d) What is the acceleration at time $t$?
(e) When is the particle speeding up? Slowing down?
(**Hint:** The object is speeding up when it's accelerating in the same direction it's moving, and slowing down when it's accelerating in the opposite direction.)
(f) What is the **total** distance the object travels during the first 8 seconds?

**4.** A baseball diamond is a square with side length 90 ft. Jason hits the ball and runs from home plate toward first base at a speed of 26 ft/s. At the moment he is 30 ft from first base,
(a) at what rate is his distance from second base decreasing?
(b) at what rate is his distance from third base increasing?




**5.** A Ferris wheel with a radius of 10 m is rotating at a rate of one revolution every 2 minutes. With your group, determine how fast a rider is *rising vertically* when his seat is 16 m above ground level. (Remember, draw a picture first!)

**Challenge Problem:** A plane flying at a constant speed of 300 km/hr passes over a ground radar station at an altitude of 1km and climbs at an angle of 30°. At what rate is the distance from the plane to the radar station increasing one minute later?

(**Hint:** You'll need the law of cosines, which says that if $a$, $b$, and $c$ are the sides of a triangle, and $\theta$ is the angle opposite the side of length $c$, then $c^2 = a^2 + b^2 - 2ab\cos\theta$.)