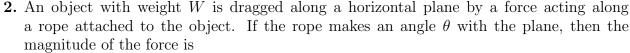
## MATH161 Workshop 5: Chain Rule; Implicit Differentiation

**Problem Set Instructions:** Work through the following problems with your group. You might not finish all of the problems, but be sure to work on all of them together and gain a good idea of how to proceed.

**1.** Find



$$F = \frac{\mu W}{\mu \sin \theta + \cos \theta}$$

where  $\mu$  is a constant called the *coefficient of friction*.

- (a) Find the rate of change of F with respect to  $\theta$ .
- (b) When is this rate of change equal to 0?

- **3.** Recall that a function f(x) is **even** if f(-x) = f(x) and **odd** if f(-x) = -f(x).
  - (a) If f(x) is an even function, is f'(x) even, odd, or neither? Hint: apply the chain rule to f(-x).
  - (b) If f(x) is an odd function, is f'(x) even, odd, or neither?

- 4. Let r be a constant. For what value(s) of r does the function  $y = e^{rx}$  satisfy the differential equation y'' 4y' + y = 0?
- 5. (a) Check that the derivatives of  $\ln(x)$  and  $\ln(2x)$  are the same. Can you explain why these two functions should have the same derivative?
  - (b) Use implicit differentiation to show that  $\frac{d}{dx}\log_a(x) = \frac{1}{x\ln a}$ . Hint: If  $y = \log_a(x)$ , then  $a^y = x$ .
  - (c) Explain the difference between  $\cos^{-1} x$  and  $(\cos x)^{-1}$ .
  - (d) Use implicit differentiation to show that  $\frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$ . Also find  $\frac{d}{dx}((\cos x)^{-1})$ .

**6.** Let h, k, and r be constants, and consider the equation

$$(x-h)^{2} + (y-k)^{2} = r^{2}.$$

- (a) Help your scribe sketch the graph of the equation. Label your graph in particular, show how the constants h, k, and r are relevant to the graph.
- (b) By just looking at the graph, determine at which points the tangent line is horizontal and at which points the tangent line is vertical.
- (c) Now, find  $\frac{dy}{dx}$  and verify your answers to part (b).

## Challenge Problem:

- (a) Show that  $\csc^{-1} x = \sin^{-1} \left(\frac{1}{x}\right)$ .
- (b) Use part (a) to show  $\frac{d}{dx} \csc^{-1} x = \frac{-1}{x^2 \sqrt{1 1/x^2}} = \frac{-1}{|x|\sqrt{x^2 1}}.$
- (c) Similarly write  $\sec^{-1} x$  in terms of  $\cos^{-1}$  and find the derivative of  $\sec^{-1} x$ .