MATH161 Workshop 5: Chain Rule; Implicit Differentiation

Problem Set Instructions: Work through the following problems with your group. You might not finish all of the problems, but be sure to work on all of them together and gain a good idea of how to proceed.

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1. Find

$$\frac{d}{dx}e^{e^{e^{e^{e^{x}}}}}$$

2. An object with weight W is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle θ with the plane, then the magnitude of the force is

$$F = \frac{\mu W}{\mu \sin\theta + \cos\theta},$$

where μ is a constant called the *coefficient of friction*.

(a) Find the rate of change of F with respect to θ.

(b) When is this rate of change equal to 0?

3. Recall that a function $f(x)$ is **even** if $f(-x) = f(x)$ and **odd** if $f(-x) = -f(x)$.

(a) If $f(x)$ is an even function, is $f'(x)$ even, odd, or neither? Hint: apply the chain rule to $f(-x)$.

(b) If $f(x)$ is an odd function, is $f'(x)$ even, odd, or neither?

4. Let r be a constant. For what value(s) of r does the function $y = e^{rx}$ satisfy the differential equation $y'' - 4y' + y = 0$?

5. (a) Check that the derivatives of $\ln(x)$ and $\ln(2x)$ are the same. Can you explain why these two functions should have the same derivative?

(b) Use implicit differentiation to show that $\frac{d}{dx} \log_a(x) = \frac{1}{x \ln a}$. **Hint:** If $y = \log_a(x)$, then $a^y = x$.

(c) Explain the difference between $\cos^{-1} x$ and $(\cos x)^{-1}$.

(d) Use implicit differentiation to show that $\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$. Also find $\frac{d}{dx}((\cos x)^{-1})$.

6. Let h, k, and r be constants, and consider the equation

$$(x-h)^2 + (y-k)^2 = r^2.$$

(a) Help your scribe sketch the graph of the equation. Label your graph — in particular, show how the constants h, k, and r are relevant to the graph.

(b) By just looking at the graph, determine at which points the tangent line is horizontal and at which points the tangent line is vertical.

(c) Now, find $\frac{dy}{dx}$ and verify your answers to part (b).

Challenge Problem:

(a) Show that $\csc^{-1} x = \sin^{-1}\left(\frac{1}{x}\right)$.

(b) Use part (a) to show $\frac{d}{dx} \csc^{-1} x = \frac{-1}{x^2\sqrt{1-1/x^2}} = \frac{-1}{|x|\sqrt{x^2-1}}$.

(c) Similarly write $\sec^{-1} x$ in terms of $\cos^{-1}$ and find the derivative of $\sec^{-1} x$.