

MTH161 Workshop 4: Derivative Basics; Product/Quotient Rules

Problem Set Instructions: Work through the following problems with your group. You might not finish all of the problems, but be sure to work on all of them together and gain a good idea of how to proceed.

1. Using the fact that $\lim_{t \rightarrow 0} \frac{e^t - 1}{t} = 1$, show (using the definition of the derivative) that

$$\frac{d}{dx} e^x = e^x.$$

2. For which values of a and b is the line $2x + y = b$ tangent to the parabola $y = ax^2$ at $x = 2$?

3. The temperature T , in degrees Fahrenheit, of a cold potato placed in a hot oven is given by $T = h(t)$, where t is the time in minutes since the potato was put into the oven.
- Discuss with your group what $h(20) = 125$ means in the context of this application.
 - Discuss with your group what $h'(20) = 2.3$ means in the context of this application.
 - Use this information to *estimate* the temperature of the potato 21 minutes after it was put in the oven.
 - What do you think happens to $h(t)$ as t goes to infinity? What about $h'(t)$? Discuss with your group.

4. Let $f(x) = \sec x$ and $g(x) = \cot x$.
- (a) Use the quotient rule with the derivatives of $\sin x$ and $\cos x$ to show $f'(x) = \sec x \tan x$.
 - (b) Similarly show $g'(x) = -\csc^2 x$.
5. (a) Find an example to show that the derivative of $f(x)g(x)$ is not equal to $f'(x)g'(x)$.
- (b) Show that if f , g , and h are differentiable functions, then $(fgh)' = f'gh + fg'h + fgh'$.
(Hint: Use the Product Rule Twice)
- (c) Find $\frac{d}{dx}(4x^3\sqrt{x}e^x \sin(x))$.

Challenge 1: Consider the function

$$f(x) = \begin{cases} x^2, & \text{if } x \text{ is rational;} \\ -x^2, & \text{if } x \text{ is irrational.} \end{cases}$$

- (a) Where is f continuous? Discontinuous?
- (b) Is $f'(0)$ defined? (You'll need to use the definition of the derivative.)

Challenge 2: Find the value of c such that the line $y = \frac{3}{2}x + 6$ is tangent to the curve $y = c\sqrt{x}$. What is the point of tangency for the c you found?