

## MTH161 Workshop 2: Limits

**Problem Set Instructions:** Work through the following problems with your group. It is ok if you do not finish all of the problems, but be sure to work on all of them together and gain a good idea of how to proceed.

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1. (a) Find functions  $f(t)$  and  $g(t)$  such that  $\lim_{t \rightarrow 0} f(t) = 0 = \lim_{t \rightarrow 0} g(t)$ , and  $\lim_{t \rightarrow 0} \frac{f(t)}{g(t)} = 5$ .
- (b) Find functions  $f(t)$  and  $g(t)$  such that  $\lim_{t \rightarrow 0} f(t) = 0 = \lim_{t \rightarrow 0} g(t)$ , and  $\lim_{t \rightarrow 0} \frac{f(t)}{g(t)} = \infty$ .
- (c) Find functions  $f(t)$  and  $g(t)$  such that  $\lim_{t \rightarrow 0} f(t) = 0 = \lim_{t \rightarrow 0} g(t)$ , and  $\lim_{t \rightarrow 0} \frac{f(t)}{g(t)} = 0$ .

2. This question pertains to the indeterminate form  $\infty - \infty$ . If the statement below is true, explain why. If the statement below is false, provide a counterexample.

**True or false:** If  $\lim_{x \rightarrow 0} f(x) = \infty$  and  $\lim_{x \rightarrow 0} g(x) = \infty$ , then  $\lim_{x \rightarrow 0} (f(x) - g(x)) = 0$ .

3. For the function

$$g(t) = \frac{1}{1 + e^{1/t}},$$

decide if  $\lim_{t \rightarrow 0} g(t)$  exists by computing the limits from the left and right separately. To help you answer this question, compute the following one-sided limits:

- |                                    |  |  |
|------------------------------------|--|--|
| (a) $\lim_{t \rightarrow 0^+} 1/t$ | (c) $\lim_{t \rightarrow 0^+} e^{1/t}$ | (e) $\lim_{t \rightarrow 0^+} \frac{1}{1 + e^{1/t}}$ |
| (b) $\lim_{t \rightarrow 0^-} 1/t$ | (d) $\lim_{t \rightarrow 0^-} e^{1/t}$ | (f) $\lim_{t \rightarrow 0^-} \frac{1}{1 + e^{1/t}}$ |

4. With your group, sketch the graph of a single function with all of the following properties:

(a)  $\lim_{x \rightarrow 0^-} f(x) = 2.$

(d)  $\lim_{x \rightarrow 4^+} f(x) = 0.$

(b)  $\lim_{x \rightarrow 0^+} f(x) = 0.$

(e)  $x = 2$  is a vertical asymptote for  $f$ .

(c)  $\lim_{x \rightarrow 4^-} f(x) = 2.$

(f)  $f(0) = 2$  and  $f(4) = 1.$

Can you draw a second graph with the same properties?

5. Consider the function  $f$  defined by

$$f(x) = \begin{cases} -1, & \text{if } x < 0; \\ 0, & \text{if } x = 0; \\ 1, & \text{if } x > 0. \end{cases}$$

(a) With your group, sketch the graph of  $y = f(x)$ .

(b) Find each of the following limits, or explain why the limit doesn't exist.

(i)  $\lim_{x \rightarrow 0^+} f(x)$

(iii)  $\lim_{x \rightarrow 0} f(x)$

(ii)  $\lim_{x \rightarrow 0^-} f(x)$

(iv)  $\lim_{x \rightarrow 0} |f(x)|$

6. What is  $\lim_{x \rightarrow 0} (e^x - 1)^2 \cos(1/x^2)$ ? Fully justify your answer. What theorem did you use?