MTH161 Workshop 1: trigonometry, transformations of functions, inverses

---

Problem Set Instructions: Work through the following problems with your group. It is ok if you do not finish all of the problems, but be sure to work on all of them together and gain a good idea of how to proceed.

---

(1) (a) Use Pythagorean identity and the cosine double angle formula

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

to write $\cos^2(x)$ in terms of $\cos(2x)$ but no other trig functions. Similarly write $\sin^2(x)$ in terms of $\cos(2x)$ but no other trig functions.

(b) The sine, cosine addition formulas are given by

$$\begin{aligned} \sin(x \pm y) &= \sin(x)\cos(y) \pm \cos(x)\sin(y) \\ \cos(x \pm y) &= \cos(x)\cos(y) \mp \sin(x)\sin(y) \end{aligned}$$

Use these to write $\sin(x)\sin(y), \cos(x)\cos(y)$, and $\sin(x)\cos(y)$ in terms of $\sin(x+y), \sin(x-y), \cos(x+y)$, and $\cos(x-y)$.

(2) Consider the following questions with your group.

(a) Sketch the graph of $\cos(x)$

(b) Starting with the graph of $\sin(x)$, use a series of graph transformations to sketch the graph of $\sin\left(\frac{\pi}{2} - x\right)$. How does it compare to the graph of $\cos(x)$?

(c) By drawing an appropriate right triangle, convince yourselves that if θ is an **acute** angle (i.e. $\theta \in (0, \pi/2)$), then $\cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right)$.

(d) Use the trigonometric identity $\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ to prove that $\cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right)$ for all $\theta \in \mathbb{R}$.

(3) With your group, consider the function

$$f(x) = \ln(|x-4|) - 1.$$

(a) Find the domain and range of f.

(b) Sketch the graph of f. **Hint:** Start with the graph of $\ln(x)$

(4) With your group:

(a) Simplify the expression $\cot(\arcsin(z))$.

(b) Find the exact value of the expressions $\cos(2\sin^{-1}(5/13))$

(c) Find the exact value of the expressions $\sin(2\sin^{-1}(5/13))$.

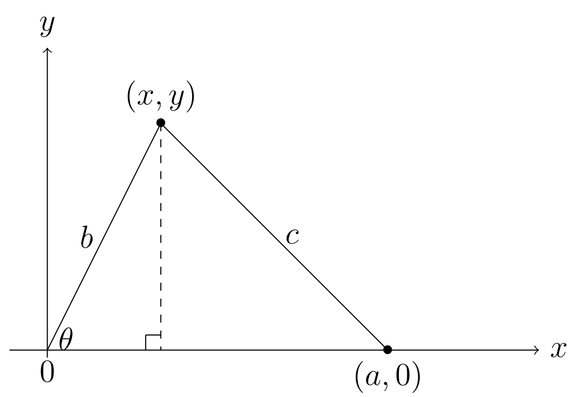
(5) Consider the following function with your group:

$$f(x) = \begin{cases} -x^2 - 1, & \text{if } x < 0; \\ e^x - 2, & \text{if } x \geq 0. \end{cases}$$

(a) Help your scribe sketch the graph of $f(x)$.
(b) Discuss with each other why the function is one-to-one and why the range is $\mathbb{R}$. (Use your sketch of the graph.)
(c) Help your scribe sketch $f^{-1}(x)$ without working out what the function f^{-1} actually is.
(d) Together, write out $f^{-1}(x)$ as a piecewise-defined function.
Hint: Find the inverse of each component of $f(x)$, keeping in mind their restricted domains and ranges.

(6) **Law of Cosines**: If a triangle has sides with lengths a, b, and c, and θ is the angle between the sides with lengths a and b, then

$$c^2 = a^2 + b^2 - 2ab\cos\theta.$$



With your group, prove the Law of Cosines assuming $0 < \theta < \pi/2$. **Hint**: Use two different triangles to write y in two different ways in terms of a, b, c, and x.

Challenge problem: What if we assume $\pi/2 < \theta < \pi$? How does this change your solution?