Math 161

Midterm 1

February 25, 2016

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Student ID Number: ______________________

Circle your instructor: **Bridy (MW 2:00)** **Demiroglu (MW 4:50)**

Academic honesty statement:
With my signature, I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: ______________________ Date: __________

- **Justify your answers.**
- **No calculators are allowed on this exam, but you are allowed one sheet of paper with writing on both sides.**
- **You do not need to simplify arithmetic expressions like 5^8 or $\frac{24}{120} + \frac{14}{36}$, but you do need to evaluate expressions like $\sin^{-1}(1)$, $\sin(\pi)$, or $e^{\ln 2}$.**

QUESTION	VALUE	SCORE
1	15	
2	15	
3	15	
4	20	
5	10	
6	15	
7	10	
TOTAL	100	

1. (15 points)

5 (a) Let $f(x) = \sin(3x)$ and $g(x) = \dfrac{x}{2}$. Compute $(f \circ g)(\pi)$ and $(g \circ f)(\pi)$.

$(f \circ g)(\pi) = f(g(\pi)) = f\left(\frac{\pi}{2}\right) = \sin \frac{3\pi}{2} = -1$

$(g \circ f)(\pi) = g(f(\pi)) = g(\sin 3\pi) = g(0) = \frac{0}{2} = 0$

5 (b) Let $f(x) = 5^{2x-3}$. Find a formula for $f^{-1}(x)$.

$y = 5^{2x-3}$

$5^3 \cdot y = 5^{2x-3} \cdot 5^3$

$125y = 5^{2x}$

$\log_5(125y) = \log_5 5^{2x}$

$\log_5(125y) = 2x$

$\dfrac{\log_5(125y)}{2} = x$

$x = \log_5 \sqrt{125y}$

interchange x and y

$y = f^{-1}(x) = \log_5 \sqrt{125x}$

5 (c) Solve $2^{2x^2+1} = 8$.

$2^{2x^2+1} = 2^3$

$2x^2 + 1 = 3$

$2x^2 = 2$

$x^2 = 1$

$|x| = 1$

$x = 1$ or $x = -1$

$\{1, -1\}$

2. **(15 points)**

5 (a) Find all solutions to the inequality $|4x-6| \geq 14$. Write your answer as an interval or as a union of intervals.

$4x-6 \geq 14$ OR $4x-6 \leq -14$

$4x \geq 20$ $\qquad$ $4x \leq -8$

$x \geq 5$ $\qquad$ $x \leq -2$

Answer $= (-\infty, -2] \cup [5, +\infty)$

5 (b) Write $\tan(\underbrace{\sin^{-1}(2x)}_{\theta})$ in terms of x in a way that has no trig or inverse trig functions.

$\theta = \sin^{-1}(2x) \iff \sin\theta = 2x$ and $\frac{-\pi}{2} \leq \theta \leq \frac{\pi}{2}$.

We know $\sin^2\theta + \cos^2\theta = 1$, so we have $(2x)^2 + \cos^2\theta = 1$

$\cos^2\theta = 1 - 4x^2$

$\cos\theta = \pm\sqrt{1-4x^2} \implies \cos\theta = +\sqrt{1-4x^2}$

but remember

$-\pi/2 \leq \theta \leq \pi/2$

Answer $= \tan\theta = \dfrac{\sin\theta}{\cos\theta} = \boxed{\dfrac{2x}{\sqrt{1-4x^2}}}$

5 (c) Solve $\ln(x+2) + \ln(x) - \ln(3) = 0$.

$\ln\left[\dfrac{x(x+2)}{3}\right] = 0$

$\dfrac{x.(x+2)}{3} = 1$

$x^2 + 2x = 3$

$x^2 + 2x - 3 = 0$

$x \quad +3$

$x \quad -1$

$(x+3)(x-1) = 0$

$x = -3$ or $x = 1$

But notice that $x = -3$ is not a solution, since $\ln(-3)$ doesn't make sense!

Answer $= \{1\}$.

3. (15 points)

10 (a) Use the definition of the derivative to compute $f'(2)$, where

$$f(x) = \frac{x}{x+3}.$$

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{\frac{2+h}{2+h+3} - \frac{2}{5}}{h} =$$

$$= \lim_{h \to 0} \frac{\frac{2+h}{5+h} - \frac{2}{5}}{h} = \lim_{h \to 0} \frac{10+5h-2(5+h)}{5 \cdot (5+h) \cdot h} =$$

$$= \lim_{h \to 0} \frac{\cancel{10}+5h-\cancel{10}-2h}{5(5+h) \cdot h} = \lim_{h \to 0} \frac{3\cancel{h}}{5(5+h) \cdot \cancel{h}} =$$

$$= \lim_{h \to 0} \frac{3}{5 \cdot (5+h)} = \frac{3}{25}$$

5 (b) Suppose you know that $f'(-2) = 3$ (you don't have to show this). Write an equation for the tangent line to $y = f(x)$ at the point where $x = -2$.

$$f(-2) = y = \frac{-2}{-2+3} = \frac{-2}{1} = -2$$

$m = f'(-2) = 3$ at the pnt. $(-2, -2)$

$$\frac{y-(-2)}{x-(-2)} = 3 \implies 3(x+2) = y+2$$

$$3x+6 = y+2$$

$$3x - y + 4 = 0$$

4. (20 points) Compute the following limits. If the limit does not exist, write "DNE." Be sure to distinguish between limits that are ∞ or $-\infty$ instead of "DNE." You may only use methods discussed in this class so far.

5 (a) $\lim_{x\to 3^-} \frac{2x^2-18}{x^2-6x+9}$ $= \lim_{x \to 3^-} \frac{2(x-3)(x+3)}{(x-3)(x-3)} =$

$= \lim_{x \to 3^-} \frac{2(x+3)}{x-3} = -\infty$ (D.N.E)

when $x \to 3^-$, the denominator becomes neg. and very, very small; and the nominator gets closer to 12.

5 (b) $\lim_{x\to 2} \frac{x^3-4x}{x^2+5x-14}$ $= \lim_{x \to 2} \frac{x(x^2-4)}{(x+7)(x-2)} =$

$= \lim_{x \to 2} \frac{x \cdot \cancel{(x-2)}(x+2)}{(x+7)\cancel{(x-2)}} = \frac{2 \cdot 4}{9} = \frac{8}{9}$

5

(c) $\lim_{x\to 3} \dfrac{2x-6}{|x-3|}$

$$\lim_{x\to 3^+} \frac{2x-6}{|x-3|} = \lim_{x\to 3^+} \frac{2(x-3)}{x-3} = 2$$

$$\lim_{x\to 3^-} \frac{2x-6}{|x-3|} = \lim_{x\to 3^-} \frac{2(x-3)}{-(x-3)} = -2$$

Since right limit $\neq$ left limit, limit DNE!

5

(d) $\lim_{x\to\infty} \sqrt{4x^2+5x} - 2x \quad = \infty - \infty$ indeterminate.

Multiply and divide w/ so-called conjugate =

$$\lim_{x\to+\infty} \left(\sqrt{4x^2+5x} - 2x\right) \cdot \left(\frac{\sqrt{4x^2+5x} + 2x}{\sqrt{4x^2+5x} + 2x}\right) =$$

$$= \lim_{x\to+\infty} \frac{4x^2+5x-4x^2}{\sqrt{4x^2+5x} + 2x} = \lim_{x\to+\infty} \frac{5x}{|2x| \cdot \sqrt{1+\frac{5}{4x}} + 2x}$$

$$= \lim_{x\to+\infty} \frac{5x}{2x\left(\sqrt{1+\frac{5}{4x}} + 1\right)} =$$

$$= \lim_{x\to+\infty} \frac{5}{2\left(\underbrace{\sqrt{1+\frac{5}{4x}}}_{1} + 1\right)} = \frac{5}{2\cdot(1+1)} = \frac{5}{4}.$$

5. **(10 points)** Find all vertical and horizontal asymptotes of $f(x) = \dfrac{\sqrt{9x^2}+x}{x}$.

$$f(x) = \frac{|3x|+x}{x}$$

To find hor. asymptotes :

$$\lim_{x\to+\infty} \frac{|3x|+x}{x} = \lim_{x\to+\infty} \frac{4x}{x} = 4$$

$$\lim_{x\to-\infty} \frac{|3x|+x}{x} = \lim_{x\to-\infty} \frac{-2x}{x} = -2$$

So, $y=4$ and $y=-2$ are hor. asymptotes.

Now notice that if $x>0$ $f(x) = \frac{4x}{\cancel{x}} = 4$

if $x<0$ $f(x) = \frac{-2x}{x} = -2$

if $x=0$, then $f(x)$ is not defined.

So, there ~~isn't~~ aren't any vertical asymptotes,

since there ~~is~~ aren't any $x=a$ s.t. the func. blows up around it.

6. (15 points)

5 (a) State the definition of continuity of the function $f(x)$ at the point a.

$f(x)$ is cont. at $x = a$ iff $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = f(a)$

10 (b) Let the following piecewise function be defined for some numbers a, b, and c.

$$f(x) = \begin{cases} x^2 + 3 & : x < -2 \\ a & : x = -2 \\ bx + c & : -2 < x < 1 \\ 2x^3 - 1 & : x \geq 1 \end{cases}$$

Find the values of a, b, c that make $f(x)$ continuous everywhere. (Justify your answer.)

$(-2)^2 + 3 = a$

$4 + 3 = a$

$\boxed{7 = a}$

$7 = -2b + c$

And $b \cdot 1 + c = 2 \cdot 1^3 - 1$

$b + c = 2 - 1$

$b + c = 1$

So. $7 = -2b + \cancel{c}$

$+ \quad b + \cancel{c} = 1$

$b + 7 = 1 - 2b$

$3b = -7 + 1$

$3b = -6$

$\boxed{b = -2}$

$\boxed{c = 3}$

7. (10 points) Suppose you have a function $f(x)$ and you know that $f'(x) = \dfrac{x^2+3x}{x^2-2x+1}$.

(a) Find all values of x where the tangent line to the graph of $y = f(x)$ is perpendicular to the line $y + x + 1 = 0$.

5

$y = -x - 1$

$m_{\ell_1} = -1$

$m_T = 1$ $\longrightarrow$ So, we have

Since $m_{\ell_1} \perp m_T$

$m_{\ell_1} \cdot m_T = -1$

$$\frac{x^2+3x}{x^2-2x+1} \neq 1$$

$$x^2+3x = x^2-2x+1$$

$$5x = 1$$

$$\boxed{x = \frac{1}{5}}$$

5

(b) Is $f(x)$ continuous at $x = 2$? ~~Is $f(x)$ continuous at $x = 1$?~~

Yes, since $f'(2) = \dfrac{4+6}{4-4+1} = 10$ that means f is

2 pnt.s

diff'ble at $x = 2$ and the derivative is 10.

So, f has to be cont. at $x = 2$, since we

know diff'bility $\Rightarrow$ continuity.