MTH 161

Final Exam

Sunday, December 17, 2017

Last Name (Family Name) ______________________________

First Name (Given Name) ______________________________

Student ID Number: ______________________________

Circle your instructor and class time:

Hambrook (MW 10:25) Hambrook (MW 2:00)

Lorman (TuTh 9:40) Lubkin (MW 9:00) Xi (TuTh 3:25)

Please read the following instructions very carefully:

- Only pens and pencils are allowed. The presence of notes, calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden.
- This exam has 15 problems and 24 pages. Check that your exam is complete when you start.
- Show your work. You may not receive full credit if insufficient justification is given.
- Clearly circle or label your final answers.
- If you need extra space, use the back of the opposite page, and write that you are doing so.
- Sign the following academic honesty statement: *I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.*

Signature: ______________________________

Part A		
QUESTION	VALUE	SCORE
1	20	
2	16	
3	16	
4	12	
5	14	
6	14	
7	8	
TOTAL	100	

Part B		
QUESTION	VALUE	SCORE
8	8	
9	12	
10	8	
11	15	
12	20	
13	10	
14	12	
15	15	
TOTAL	100	

Part A

1. (20 points)

(a) If $f(x) = \dfrac{x+1}{2x+1}$, find a formula for $f^{-1}(x)$.

(b) Solve $\ln x + \ln(x-1) = \ln 6$.

(c) Find the exact value of $\tan\left(\sin^{-1}\left(\frac{-1}{9}\right)\right)$.

(d) Solve $|x| + |2x - 1| \geq 7$.

2. (16 points) Compute the derivative (with respect to x) of each of the following functions.

(a) $\dfrac{x^3}{\cos(x^3)}$

(b) $e^{\sqrt{\ln x}}$

(c) $\sqrt[3]{1+x^2} + \dfrac{1}{\sqrt{1+x^3}}$

(d) $(\ln x)^{\ln x}$

3. (16 points) Evaluate the following limits.

(a) $\lim\limits_{x \to 1^+} \dfrac{x^2 - 9}{x^2 + 2x - 3}$

(b) $\lim\limits_{x \to -\infty} \dfrac{\sqrt{x^6 + 1}}{(2x + 1)^3}$

(c) $\lim_{x \to 0} \dfrac{\sqrt{ax + b^2} - b}{x}$ (where a and b are positive constants)

(d) $\lim_{x \to 0} \dfrac{|2x - 1| - |2x + 1|}{x}$

4. (12 points) Consider the curve $y \sin 2x = x \cos 2y$.

(a) Find $\dfrac{dy}{dx}$ at the point $(\pi/2, \pi/4)$.

(b) Find an equation for the tangent line to the curve at the point $(\pi/2, \pi/4)$.

5. (14 points) A plane is climbing at an angle of 30° while flying at a constant speed of 300 km/h. It passes over a ground radar station at an altitude of 7 km. At what rate is the distance from the plane to the radar station increasing 1 minute later?

6. (14 points) The height of the foam in a glass of (root) beer decreases at a rate proportional to the current height. The glass is filled with (root) beer so that the top 5 cm is foam. After 60 seconds, only 2 cm of foam remains.

(a) Find an expression for the height of the foam t seconds after the (root) beer is poured.

(b) At what time is the height of the foam 4 cm?

(c) How long must we wait until the foam completely disappears?

7. (8 points) For each statement below, circle T (true) if the statement is always true. Otherwise, circle F (false)

(a) T or F If f is continuous on (a, b) and $f(a) < 0 < f(b)$, then there is a number c in (a, b) such that $f(c) = 0$.

(b) T or F If $f'(0) = 5$, then $\lim_{h \to 0} \frac{f(h)}{h} = 5$.

(c) T or F If $f(1) = g(1)$ and $f'(x) \leq g'(x)$ for all x in $[0, 1]$, then $f(0) \geq g(0)$.

(d) T or F $f(x) = x|x|$ is differentiable at every real number x.

Part B

8. (8 points) For each statement below, circle T (true) if the statement is always true. Otherwise, circle F (false)

(a) T or F If f is continuous, then $\int_0^5 f(x)dx \le \int_0^5 |f(x)|dx$

(b) T or F If f is continuous, then $\int_0^5 f(x)dx \le \int_0^5 (f(x))^2 dx$

(c) T or F If f is continuous, then $\int_0^5 f(x)dx \le \int_0^5 (f(x)+2)dx$

(d) T or F If f is continuous, then $\int_0^5 f(x)dx \le \int_0^{10} f(x)dx$

9. (12 points)

(a) Let $f(x)$ be an increasing function. If a Riemann sum with right endpoints is used to approximate $\int_0^1 f(x)dx$, must the Riemann sum be larger than the integral? Justify your answer with an appropriate sketch.

(b) Let $f(x)$ be an increasing function and let $L(x) = f'(1)(x-1) + f(1)$ be the linear approximation function of $f(x)$ at 1. Must $L(1.01)$ be larger than $f(1.01)$? Justify your answer with an appropriate sketch.

10. (8 points) Evaluate the following limits.

(a) $\displaystyle\lim_{x\to 0} \frac{\sqrt{1+2x} - \sqrt{1-4x}}{x}$

(b) $\displaystyle\lim_{x\to 0^+} (\tan 2x)^x$

11. (15 points) Evaluate the following integrals.

(a) $\displaystyle\int_1^{e^2} \frac{\sqrt{\ln x + 1}}{x} dx$

(b) $\displaystyle\int \frac{x^5}{\sqrt{x^3+1}} dx$

(c) $\int_0^3 |e^x - 2| dx$

12. (20 points) Consider the function f with its first and second derivatives:

$$f(x) = \frac{1}{\sqrt[3]{x^3+1}}, \quad f'(x) = \frac{-x^2}{\left(\sqrt[3]{x^3+1}\right)^4}, \quad f''(x) = \frac{2x(x^3-1)}{\left(\sqrt[3]{x^3+1}\right)^7}.$$

(a) Find the domain of $f(x)$.

(b) List all x-intercepts and y-intercepts of $f(x)$.

Reminder: $f(x) = \dfrac{1}{\sqrt[3]{x^3+1}}, \quad f'(x) = \dfrac{-x^2}{\left(\sqrt[3]{x^3+1}\right)^4}, \quad f''(x) = \dfrac{2x(x^3-1)}{\left(\sqrt[3]{x^3+1}\right)^7}.$

(c) Compute $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ for any vertical asymptotes $x = a$.

(d) Compute $\lim_{x \to -\infty} f(x)$ and $\lim_{x \to \infty} f(x)$. List all horizontal asymptotes of $f(x)$.

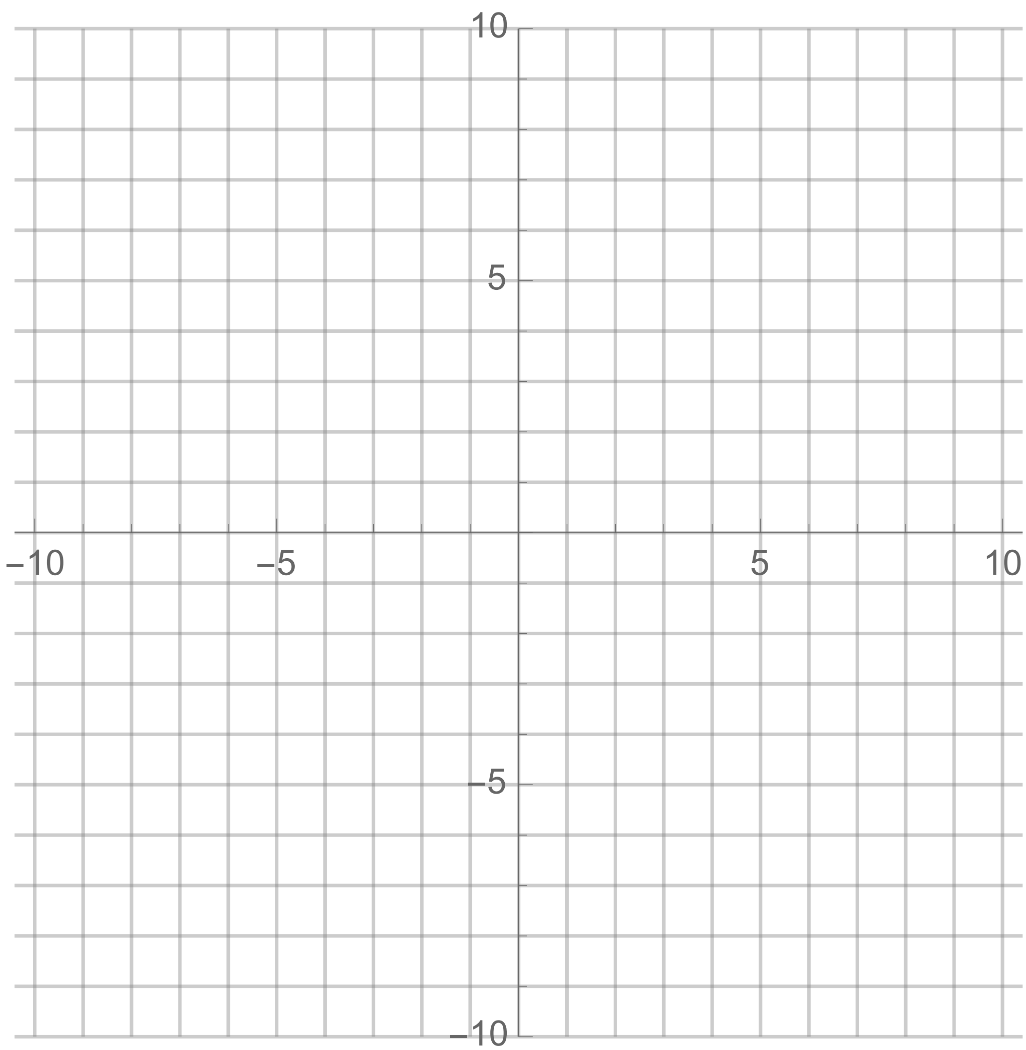
Reminder: $f(x) = \dfrac{1}{\sqrt[3]{x^3+1}}, \quad f'(x) = \dfrac{-x^2}{\left(\sqrt[3]{x^3+1}\right)^4}, \quad f''(x) = \dfrac{2x(x^3-1)}{\left(\sqrt[3]{x^3+1}\right)^7}.$

(e) On what intervals is $f(x)$ increasing? decreasing?

(f) On what intervals is $f(x)$ concave up? concave down?

13. (10 points) Sketch the graph of a function $f(x)$ that satisfies the following properties:

- x-intercepts: $-3, 3$
- y-intercepts: 2
- vertical asymptotes: $x = -5$ and $x = 5$
- horizontal asymptotes: $y = -1$ and $y = 1$
- $f'(x) > 0$ on $(-\infty, -5) \cup (-5, 0) \cup (5, \infty)$
- $f'(x) < 0$ on $(0, 5)$
- $f''(x) > 0$ on $(-\infty, -5)$
- $f''(x) < 0$ on $(-5, 5) \cup (5, \infty)$



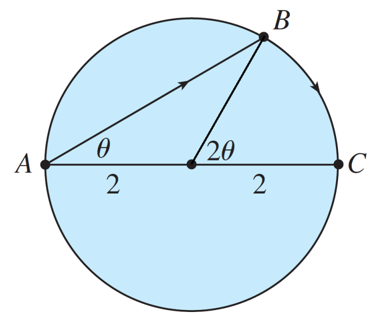
14. (12 points)

(a) A particle is moving with the given velocity and position data. Find the position function $s(t)$ of the particle.

$$v(t) = 10\sin t + 3\cos t, \quad s(\pi/4) = 12$$

(b) Let $f(x) = \int_1^{x^2} (t-4)e^{-t^2}\,dt$ for all real numbers x. On what intervals is $f(x)$ an increasing function?

15. (15 points) A woman at a point A on the shore of a circular lake with radius 2 miles wants to arrive at the point C diametrically opposite A on the other side of the lake in the shortest possible time (see the figure). She can walk at the rate of 4 miles/h and row a boat at 2 miles/h. How should she proceed? Justify your answer. (It may help to know that $\sqrt{3} = 1.73\ldots.$)



Formula Sheet

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$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$
$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$
$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\begin{aligned} \sin 2x &= 2 \sin x \cos x \\ \cos 2x &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x \end{aligned}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$
$$\sin^2 x = \frac{1-\cos 2x}{2}$$

Scratch Paper. You may tear this page off. Nothing you write on this page will be graded.