MTH 141 and 161 Basic Skills Exam

September 15, 2022

NAME (please print legibly): Solutions

Your University ID Number: ______________________

Pledge of Honesty

I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.

Signature: ______________________

Enter your answers where indicated in order to receive credit. Calculators and notes are not permitted. If you are confused about the wording of a question or need a clarification, you should raise your hand and **ask a proctor** about it.

1. (12 points) For this problem, justification is not required and partial credit will **not** be awarded. Decide which entry (if any) is equivalent to the given expression:

(a) $\dfrac{2x}{2x+y} =$

$$= \frac{\frac{1}{2x}}{\frac{1}{2x}}\left(\frac{2x}{2x+y}\right)$$

$$= \frac{1}{1+\frac{y}{2x}}$$

- ☐ $\dfrac{1}{y}$
- ☐ $1 - \dfrac{2x}{y}$
- ☐ $\dfrac{x}{x+2y}$
- ■ $\dfrac{1}{1+\dfrac{y}{2x}}$
- ☐ None of the above

(b) $\sqrt{x^2+9} =$

Important:

$$\sqrt{a^2+b^2} \neq \sqrt{a^2} + \sqrt{b^2}$$

- ☐ $|x+3|$
- ☐ $(x-3)(x+3)$
- ☐ $\dfrac{x}{\sqrt{x^2+9}}$
- ☐ x^4+81
- ■ None of the above

(c) $\dfrac{3x}{3y/z} =$

- ☐ $\dfrac{3x+z}{3y}$
- ■ $\dfrac{xz}{y}$
- ☐ $\dfrac{9xz}{y}$
- ☐ $\dfrac{x}{9yz}$
- ☐ None of the above

$= 3x \cdot \dfrac{1}{3y/z}$

$= \cancel{3}x \cdot \dfrac{z}{\cancel{3}y}$

$= \dfrac{xz}{y}$

2. (8 points) A line L contains the points (c, d) and (e, f). Determine the y-intercept of L in terms of c, d, e and f. Show your work and put your answer in the answer box. (You may assume $c \neq e$.)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f-d}{c-e}$$

Slope Intercept:

$$y = \left(\frac{f-d}{c-e}\right)x + b$$

$\rightarrow$ Using the point (c,d),

$$d = \left(\frac{f-d}{c-e}\right)c + b$$

$$b = d - c\left(\frac{f-d}{c-e}\right)$$

$\rightarrow$ using the point (e,f)

$$b = f - e\left(\frac{f-d}{c-e}\right)$$

Pt Slope

$$y - y_0 = m(x - x_0)$$

using (c,d)

$$y - d = \left(\frac{f-d}{c-e}\right)(x-c).$$

If $x = 0$, then

$$y - d = \left(\frac{f-d}{c-e}\right)(-c)$$

$$y = d - c\left(\frac{f-d}{c-e}\right)$$

using (e,f)

$$y - f = \left(\frac{f-d}{c-e}\right)(-e)$$

$$y = f - e\left(\frac{f-d}{c-e}\right)$$

Answer:

$\left(0, d - c\left(\frac{f-d}{c-e}\right)\right)$ or $\left(0, f - e\left(\frac{f-d}{c-e}\right)\right)$

3. (10 points) For each part, show your work and put your answer in the answer box.

(a) Find all x satisfiying the inequality:

$$2(x-1)^2(x+6) \leq 0.$$

Let $f(x) = 2(x-1)^2(x+6)$.

Then $f(1) = 0$ and $f(-6) = 0$.

Since $2(x-1)^2 \geq 0$ for any x, we need only consider if $x < -6$ or if $x > -6$.

If $x < -6$, then $x+6 < -6+6 = 0$.

If $x > -6$, then $x+6 > -6+6 = 0$

So if $x < -6$, $f(x) = 2\underset{+}{(x-1)^2}\underset{-}{(x+6)} < 0$.

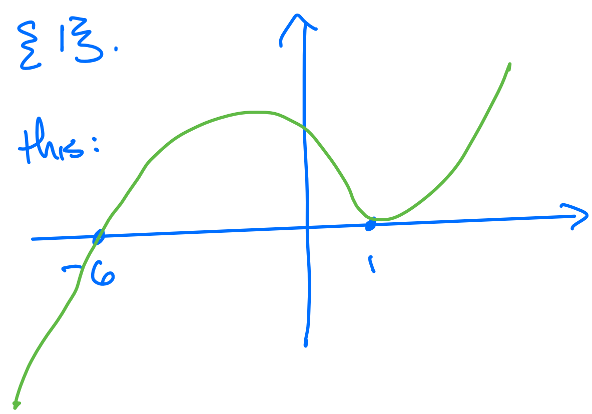
If $x > -6$, $f(x) = 2\underset{+}{(x-1)^2}\underset{+}{(x+6)} > 0$ (Except if $x=1$.)

So $f(x) < 0$ if x is in the interval $(-\infty, -6)$.

Finally, $f(x) \leq 0$ on $(-\infty, -6] \cup [1,1]$

or $(-\infty, -6] \cup \{1\}$.

Something like this:



Answer: $(-\infty, -6] \cup \{1\}$

(b) Solve for x:

$$|x-3| = |x|.$$

Case 1: $x-3 \geq 0$, $x \geq 3$, so

both $|x-3| = x-3$

and $|x| = x$

Case 2: $0 \leq x \leq 3$

then $|x-3| = -(x-3)$

and $|x| = x$

Case 3: $x \leq 0$. then $x \leq 3$, so

$|x-3| = -(x-3)$

and $|x| = -x$.

Finally:

Case 1: $x-3 = x$.

$-3 = 0$

No Solutions

Case 2: $-(x-3) = x$

$3 = 2x$

$x = \frac{3}{2}$

Case 3: $-(x-3) = -x$

$x-3 = x$

No solutions

Answer:

$x = \frac{3}{2}$

4. (16 points) For each part, show your work and put your answer in the answer box.

(a) Evaluate $e^{2\ln(7)}$ in simplest form.

Answer:

49

$$e^{2\ln 7} = e^{\ln(7^2)} = 49$$

(b) Find an integer x satisfying: $\frac{200}{x\sqrt{x}} - \frac{75}{x\sqrt{x}} = 1$.

Answer:

25

$$\frac{200 - 75}{x\sqrt{x}} = 1$$

$$125 = x\sqrt{x} = x^{\frac{3}{2}}$$

$$(125)^{\frac{1}{3}} = \left(x^{\frac{3}{2}}\right)^{\frac{1}{3}}$$

$$5 = x^{\frac{1}{2}} = \sqrt{x}$$

$$x = 25$$

(c) Solve for x:

$$\log_2(x) + \log_2(x-2) = 3$$

Answer:

$x = 4$

$\log_2 x + \log_2(x-2) = \log_2 (x)(x-2) = \log_2 (x^2-2x)$

$\log_2(x^2-2x) = 3$

$2^{\log_2(x^2-2x)} = 2^3 = 8$

$x^2 - 2x = 8$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4$, $\cancel{x = -2}$

Since -2 is not in the domain of $\log_2(x-2) + \log_2(x)$, we discard that solution.

(d) Find all solutions for x: $e^{2x} - 9e^{x} + 20 = 0$.

Answer:

$e^{2x} - 9e^{x} + 20 = (e^{x} - 5)(e^{x} - 4)$.

So $e^{x} - 5 = 0$ or $e^{x} - 4 = 0$

$e^{x} = 5$

$\ln(e^{x}) = \ln 5$

5. (13 points) Let $f(x) = \dfrac{x+2}{x-3}$.

(a) (7pts) Determine its inverse, $f^{-1}(x)$.

$$y = \frac{x+2}{x-3}$$

$$xy - 3y = x + 2$$

$$xy - x = 3y + 2$$

$$x(y-1) = 3y + 2$$

$$x = \frac{3y+2}{y-1}$$

$$\boxed{f^{-1}(x) = \frac{3x+2}{x-1}}$$

(b) (6pts) Determine the domain and range of f, as well as the domain and range of f^{-1}.

the domain of f is $(-\infty, 3) \cup (3, \infty)$ = range of f^{-1}.

the domain of f^{-1} is $(-\infty, 1) \cup (1, \infty)$ = range of f.

6. (16 points) For this problem, justification is not required and partial credit will **not** be awarded. In each part, evaluate the expression and put your answer in the answer box.

(a) $\log_8(64) =$

Answer: 2

$8^2 = 64$

(b) $\log_{27}(3) =$

Answer: $\frac{1}{3}$

$27^{\frac{1}{3}} = 3$

(c) $\log_{27}\left(\frac{1}{3}\right) =$

Answer: $-\frac{1}{3}$

$27^{-\frac{1}{3}} = \frac{1}{27^{\frac{1}{3}}} = \frac{1}{3}$

(d) $\log_5(4) - \log_5(100) =$

Answer: -2

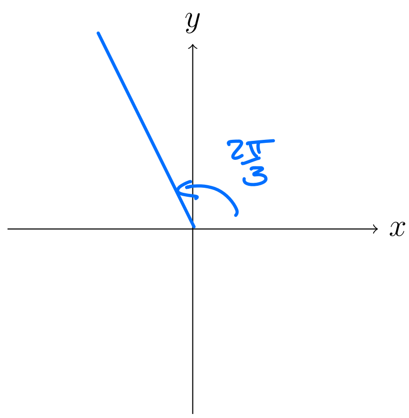
$\log_5\left(\frac{4}{100}\right)$

$= \log_5\left(\frac{1}{25}\right)$

$5^{-2} = \frac{1}{25}$

7. (15 points) For each of the following θ, draw the angle whose radian measure is θ on the axes provided. Then determine $\sin\theta$, $\tan\theta$, and $\sec\theta$.

(a) $\theta = \frac{2\pi}{3}$

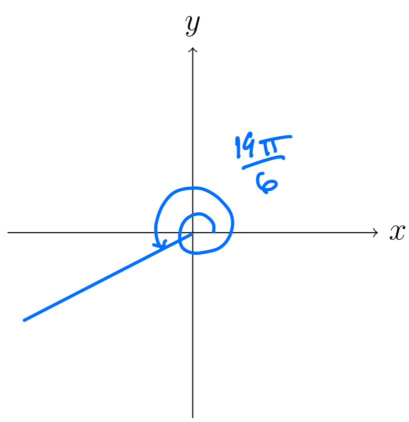


(a) $\sin\theta =$ $\frac{\sqrt{3}}{2}$

(b) $\tan\theta =$ $-\sqrt{3}$

(c) $\sec\theta =$ 2

(b) $\theta = \frac{19\pi}{6}$



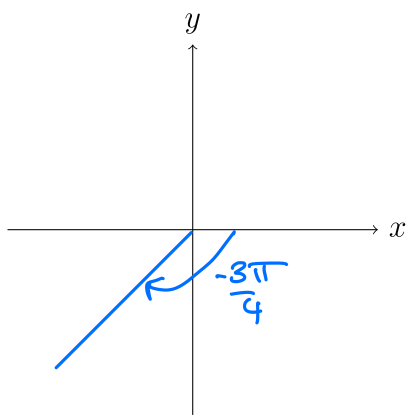
(a) $\sin\theta =$ $-\frac{1}{2}$

(b) $\tan\theta =$ $\frac{1}{\sqrt{3}}$

(c) $\sec\theta =$ $\frac{-2}{\sqrt{3}} = \frac{-2\sqrt{3}}{3}$

$$\frac{19\pi}{6} = \frac{18\pi}{6} + \frac{\pi}{6} = 3\pi + \frac{\pi}{6}$$

(c) $\theta = \dfrac{-3\pi}{4}$



(a) $\sin\theta =$ $\frac{-\sqrt{2}}{2}$

(b) $\tan\theta =$ 1

(c) $\sec\theta =$ $-\sqrt{2}$

8. (10 points) For the angle θ pictured below, find $\sin(\theta)$ and $\tan(\theta)$. Show all work.

(a) $\sin\theta =$ $\frac{-3}{\sqrt{13}}$

(b) $\tan\theta =$ $\frac{3}{2}$