

MTH 161
Midterm 1
October 13, 2022

Name: _____

UR ID: _____

Circle your Instructor's Name:

Andres Chirre

Mark Herman

Saul Lubkin

Joshua Sumpter

Emmett Wyman

Instructions:

- The presence of calculators, cell phones, and other electronic devices at this exam is strictly forbidden. Notes or texts of any kind are strictly forbidden.
- When applicable, please put your final answer in the answer box. We will judge your work outside the box as well (unless specified otherwise) so you still need to show your work and justify your answers. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given.
- In your answers, you do not need to simplify arithmetic expressions like $\sqrt{5^2 - 4^2}$. However, known values of functions should be evaluated, for example, $\ln e$, $\sin \pi$, e^0 .
- This exam is out of 100 points. You are responsible for checking that this exam has all 12 pages.

PLEASE COPY THE HONOR PLEDGE AND SIGN:

I affirm that I will not give or receive any unauthorized help on this exam, and all work will be my own.

YOUR SIGNATURE: _____

QUESTION	VALUE	SCORE
1	12	
2	12	
3	24	
4	12	
5	16	
6	12	
7	12	
TOTAL	100	

Trig Identities

- $\sin^2 \theta + \cos^2 \theta = 1$
- $\tan^2 \theta + 1 = \sec^2 \theta$
- $\cot^2 \theta + 1 = \csc^2 \theta$
- $\sin(2\theta) = 2 \sin \theta \cos \theta$
- $\sin^2 \theta = \frac{1}{2}(1 - \cos(2\theta))$
- $\cos^2 \theta = \frac{1}{2}(1 + \cos(2\theta))$
- $\sin(a + b) = \sin(a) \cos(b) + \cos(a) \sin(b)$
- $\sin(a - b) = \sin(a) \cos(b) - \cos(a) \sin(b)$
- $\cos(a + b) = \cos(a) \cos(b) - \sin(a) \sin(b)$
- $\cos(a - b) = \cos(a) \cos(b) + \sin(a) \sin(b)$
- $\sin(a) \cos(b) = \frac{1}{2}[\sin(a - b) + \sin(a + b)]$
- $\sin(a) \sin(b) = \frac{1}{2}[\cos(a - b) - \cos(a + b)]$
- $\cos(a) \cos(b) = \frac{1}{2}[\cos(a - b) + \cos(a + b)]$

1. (12 points) Answer each part below and fully justify your answers. Put your final answer in the answer box:

(a) If $|\cos \phi| = 2/3$, and $\pi/2 < \phi < \pi$, then compute all six trig functions of ϕ .

Answer:

(b) Find all x , $0 \leq x \leq 2\pi$, such that $\sin 2x = \cos x$.

Answer:

2. (12 points) Let $f(x) = \ln(e^x - 3)$. Answer each part below, show your work, and put the final answer in the answer box.

(a) Find an explicit formula for $f^{-1}(x)$.

Answer:

(b) Find the domain and range of f and f^{-1} with brief justification.

Answer:

3. (24 points) Evaluate the following limits using limit laws and properties of limits to justify your answers. If they do not exist, explain why not. If the limit is $+\infty$ or $-\infty$, state which it is. Put your final answer in the answer box.

(a) $\lim_{x \rightarrow 1^+} \frac{1 - \sqrt{x}}{x - x^2}$.

Answer:

(b) $\lim_{x \rightarrow 5^-} \frac{2}{(x - 5)^3}$.

Answer:

(c) $\lim_{x \rightarrow 0^-} \frac{|x|}{x^2} + \frac{1 - 2x^2}{x}$.

Answer:

(d) $\lim_{x \rightarrow \infty} \sin\left(\frac{1}{x}\right) \cos(x)$

Answer:

4. (12 points) Consider the following function:

$$f(x) = \begin{cases} -5 - x & \text{if } x < -1 \\ A & \text{if } x = -1 \\ x^2 + 2x - 3 & \text{if } -1 < x < 0 \\ B & \text{if } x = 0 \\ e^x + 1 & \text{if } x > 0 \end{cases}$$

(a) Compute each of the following or state why they do not exist.

$$\lim_{x \rightarrow -1^-} f(x) =$$

$$\lim_{x \rightarrow -1^+} f(x) =$$

$$\lim_{x \rightarrow -1} f(x) =$$

$$\lim_{x \rightarrow 0^-} f(x) =$$

$$\lim_{x \rightarrow 0^+} f(x) =$$

$$\lim_{x \rightarrow 0} f(x) =$$

(b) Find a value A such that $f(x)$ is continuous at $x = -1$ or explain why no such value exists. Similarly, find a value B such that $f(x)$ is continuous at $x = 0$ or explain why no such value exists. **Fully justify your answer.**

5. (16 points) Consider the following function:

$$f(x) = \frac{(x+3)\sqrt{4x^2+1}}{x^2+x-6}.$$

- (a) Determine whether or not the graph $y = f(x)$ has any vertical asymptotes. If so, state what they are. **Justify your answer using limits!**

Answer:

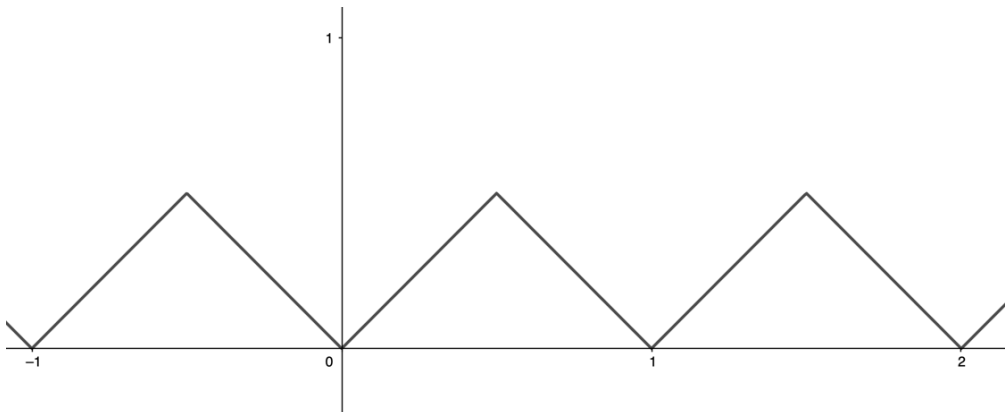
(b) Again referring to

$$f(x) = \frac{(x+3)\sqrt{4x^2+1}}{x^2+x-6},$$

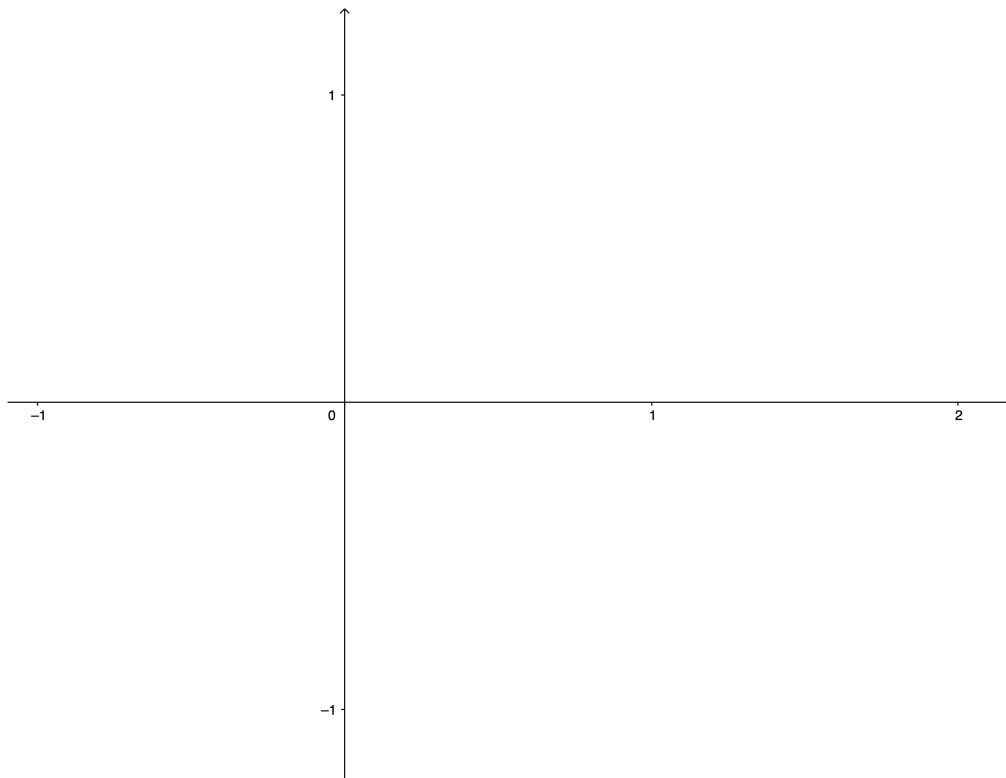
determine whether or not the graph $y = f(x)$ has any horizontal asymptotes. If so, state what they are. **Justify your answer using limits!**

Answer:

6. (12 points) Let $y = f(x)$ give the distance between x and the nearest integer, graphed below.



(a) Graph the derivative $y = f'(x)$ on the axes below.



(b) Give all real values of x in $(-\infty, \infty)$ at which f is not differentiable. Only your answer will be considered, no justification is required.

7. (12 points) Answer each part below.

(a) Write the limit definition of the derivative $f'(a)$ of a function f at a .

(b) Now let $f(x) = x^2$. Use the limit definition of the derivative to find $f'(2)$.

EXTRA PAGE. You may use this page if you run out of space. Be sure to label your problems on this page and also include a note on the original page telling the graders to look for your work here.