MTH 161

Midterm 1

Thursday, October 5, 2017

Last Name (Family Name): ____________________

First Name: ____________________

Student ID Number: ____________________

Circle your instructor and class time:

Hambrook (MW 10:25) Hambrook (MW 2:00)
Lorman (TuTh 9:40) Lubkin (MW 9:00) Xi (TuTh 3:25)

Please read the following instructions very carefully:

- You have **75 minutes** to complete this exam.
- Only pens/pencils are allowed. The presence of notes, calculators, cell phones, iPods and other electronic devices at this exam is strictly forbidden. You may refer to the **formula sheet** on the last page of this exam.
- Show your work and justify your answers. If you need extra space, use the back of the previous page and **clearly indicate** that you have done so. You may not receive full credit for a correct answer if insufficient work is shown or insufficient justification is given. **Clearly circle or label your final answers.**
- Sign the following academic honesty statement: *I affirm that I will not give or receive any unauthorized help on this exam, and that all work will be my own.*

Signature: ____________________

QUESTION	VALUE	SCORE
1	20	
2	12	
3	16	
4	20	
5	16	
6	16	
TOTAL	100	

1. (20 points)

(a) Find the solution set of the following inequality:

$$x^2 - 3x - 18 \geq 0.$$

$(x-6)(x+3) \geqslant 0.$

$\Rightarrow \quad x \geqslant 6 \quad \text{or} \quad x \leq -3.$

Solution set : $\{x \in \mathbb{R} : x \geqslant 6 \text{ or } x \leq -3\}$

or $(-\infty, -3] \cup [6, \infty)$.

(b) Find the solution set of the following inequality:

$$|x - 2| > |x|.$$

$\Leftrightarrow \quad (x-2)^2 > (x)^2.$

$\Rightarrow \quad x^2 - 4x + 4 > x^2.$

$x < 1$

Solution set: $\{x \in \mathbb{R} : x < 1\}$

or. $(-\infty, 1)$.

(c) Solve for x:

$$3^x = 5^{x-1}$$

$$\ln 3^x = \ln 5^{x-1}$$

$$x \ln 3 = x \ln 5 - \ln 5.$$

$$(\ln 5 - \ln 3)x = \ln 5$$

$$x = \frac{\ln 5}{\ln 5 - \ln 3}.$$

(d) Solve for x:

$$\ln\left(x^2 + \frac{1}{2}\right) = 0.$$

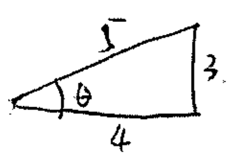
$$\Rightarrow \quad x^2 + \frac{1}{2} = 1.$$

$$x^2 = \frac{1}{2}$$

$$x = \pm\sqrt{\frac{1}{2}}.$$

(e) Find the value of:

$$\tan\left(\cos^{-1}\left(\frac{-4}{5}\right)\right).$$



let $\theta = \cos^{-1}\left(\frac{-4}{5}\right)$, $\theta \in [0, \pi]$.

then $\cos\theta = -\frac{4}{5}$, so $\theta \in [\frac{\pi}{2}, \pi]$, $\tan\theta < 0$.

$$|\tan\theta| = \frac{3}{4}.$$

$$\tan\theta = -\frac{3}{4}.$$

2. (12 points) Consider the line L given by $2x + 5y + 17 = 0$.

(a) Find the slope of this line.

$$5y = -2x - 17$$

$$y = -\frac{2}{5}x - \frac{17}{5}$$

slope $m = -\frac{2}{5}$.

(b) Find an equation for the line that is perpendicular to L and passes through the point $(0, 1)$.

Suppose this line has slope m_1.

then $m_1 \cdot m = -1 \Rightarrow m_1 = -\frac{1}{m} = \frac{5}{2}$.

thus line equation: $y = \frac{5}{2}x + 1$.

(c) Find an equation for the line that is parallel to L and passes through the point $(0, 1)$.

Slope $m_2 =$ Slope of $L = m = -\frac{2}{5}$.

$$y = -\frac{2}{5}x + 1.$$

3. (16 points) Let $f(x) = \dfrac{2x}{x+1}$, $g(x) = x^2 - 1$, and $h(x) = \sqrt{x}$.

(a) Find $(f - g + h)(4)$.

$$(f-g+h)(4)$$
$$= f(4) - g(4) + h(4)$$
$$= \frac{8}{4+1} - (16-1) + 2$$
$$= \frac{8}{5} - 13.$$

(b) Find $h \circ g(x) = h(g(x))$ and its domain.

$$h \circ g(x) = h(g(x)) = h(x^2-1). = \sqrt{x^2-1}.$$

with domain : $\{x \in \mathbb{R} : x^2 - 1 \geq 0\}$.

$= \{x \in \mathbb{R} : x \geq 1 \text{ or } x \leq -1\}$.

$= (-\infty, -1] \cup [1, \infty)$.

(c) Find $g \circ f(x) = g(f(x))$ and its domain.

$$g \circ f(x) = g(f(x)) = g\left(\frac{2x}{x+1}\right)$$

$$= \frac{4x^2}{(x+1)^2} - 1$$

with domain : $\{x \in \mathbb{R} : x \neq -1\}$.

$= (-\infty, -1) \cup (-1, \infty)$.

(d) Find $f^{-1}(x)$ and its domain.

f : $y = \frac{2x}{x+1}$, $x \neq -1$.

$$yx + y = 2x$$

$$(y-2)x = -y$$

$$x = -\frac{y}{y-2}.$$

So $f^{-1}(x) = -\frac{x}{x-2}$, $\{x \in \mathbb{R} : x \neq 2.\} = (-\infty, 2) \cup (2, \infty)$

4. (20 points) Compute the following limits.

- If the limit does not exist, write "DNE."
- When appropriate, write ∞ or $-\infty$ instead of "DNE."
- You may only use methods discussed up to this point in the course.

(a) $\lim_{x \to 4} \frac{x}{3x-8}$

as $x \to 4$, $3x-8 \to 4 \neq 0$.

So $\lim_{x \to 4} \frac{x}{3x-8} = \lim_{x \to 4} \frac{4}{4} = 1$

(b) $\lim_{x \to 2} \frac{|x-2|}{x^2-4}$

$$\lim_{x \to 2^+} \frac{|x-2|}{x^2-4} = \lim_{x \to 2^+} \frac{(x-2)}{(x-2)(x+2)} = \lim_{x \to 2^+} \frac{1}{x+2} = \frac{1}{4}.$$

$$\lim_{x \to 2^-} \frac{|x-2|}{x^2-4} = \lim_{x \to 2^-} \frac{-(x-2)}{(x-2)(x+2)} = \lim_{x \to 2^-} \frac{-1}{x+2} = \frac{-1}{4}$$

So DNE.

(c) $\lim_{x\to 0^+} \frac{2-\sqrt{x+4}}{x}$

$$= \lim_{x\to 0^+} \frac{2-\sqrt{x+4}}{x} \cdot \frac{2+\sqrt{x+4}}{2+\sqrt{x+4}}$$

$$= \lim_{x\to 0^+} \frac{4-x-4}{x(2+\sqrt{x+4})}$$

$$= \lim_{x\to 0^+} \frac{-1}{2+\sqrt{x+4}} = -\frac{1}{4}$$

(d) $\lim_{x\to 5} \frac{\frac{1}{x}-\frac{1}{5}}{x-5}$

$$= \lim_{x\to 5} \frac{\frac{5-x}{5x}}{x-5}$$

$$= \lim_{x\to 5} \frac{-1}{5x} = \frac{-1}{25}$$

(e) $\lim_{x\to 0^-} \left(\frac{1}{x} + \frac{1}{|x|}\right)$

as $x \to 0^-$, $x < 0$

so $\frac{1}{|x|} = -\frac{1}{x}$.

$\lim_{x\to 0^-} \left(\frac{1}{x} + \frac{1}{|x|}\right)$

$= \lim_{x\to 0^-} \frac{1}{x} - \frac{1}{x} = 0$.

5. (16 points) Let

$$f(x) = \begin{cases} x, & \text{if } x < 0 \\ e^x - 1, & \text{if } 0 \le x \le 2 \\ 2 - x, & \text{if } x > 2. \end{cases}$$

(a) Find the numbers at which f is discontinuous.

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x = 0$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (e^x - 1) = 0$$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (e^x - 1) = e^2 - 1$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (2 - x) = 0$$

Therefore, f is discontinuous at $x = 2$.

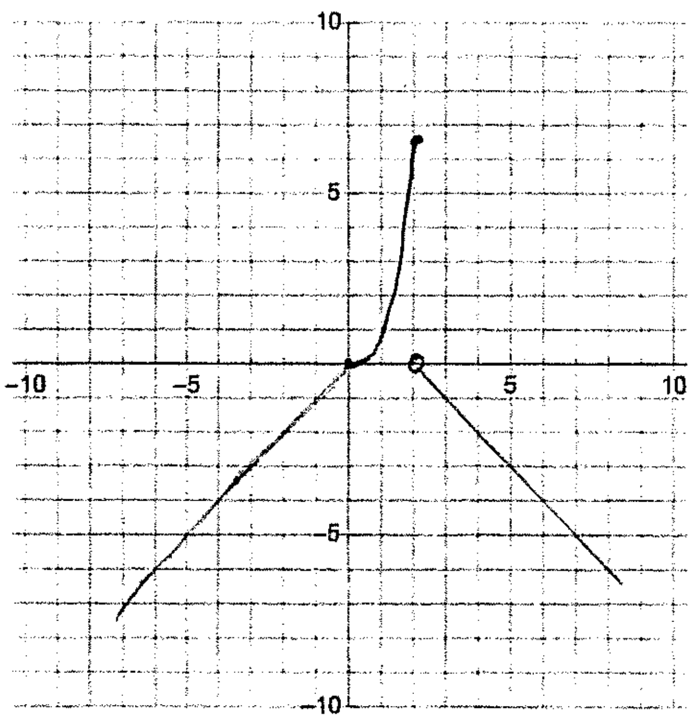
(b) At each number where f is discontinuous, determine if f is continuous is from the right, from the left, or neither.

At $x = 2$.

$f(x) = e^x - 1$, $f(2) = e^2 - 1 = \lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$

So f is continuous from the left.

(c) Sketch the graph of f below.



6. (16 points)

(a) State the Intermediate Value Theorem.

Suppose f is continuous on $[a, b]$, and let N be a number between $f(a)$ and $f(b)$ where $f(a) \neq f(b)$. Then there exists a number $c \in (a, b)$ such that $f(c) = N$.

(b) Use the Intermediate Value Theorem to show that the equation

$$\sin x = 1 - x$$

has a solution x between 0 and $\frac{\pi}{2}$.

Let $f(x) = \sin x - 1 + x$.

$f(0) = \sin 0 - 1 + 0 = -1 < 0$

$f\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2} - 1 + \frac{\pi}{2} = \frac{\pi}{2} - 1 + 1 = \frac{\pi}{2} > 0$

Since f is continuous on $[0, \frac{\pi}{2}]$ and $f(0) < 0 < f(\frac{\pi}{2})$, by I.V.T. there exists $c \in (0, \frac{\pi}{2})$ such that $f(c) = 0$

$\Rightarrow \sin c = 1 - c.$

Formula Sheet

Midterm 1

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$
$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$
$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\begin{aligned}
\sin 2x &= 2\sin x \cos x \\
\cos 2x &= \cos^2 x - \sin^2 x \\
&= 2\cos^2 x - 1 \\
&= 1 - 2\sin^2 x
\end{aligned}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$
$$\sin^2 x = \frac{1-\cos 2x}{2}$$